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Π-STRATEGY FOR A DIFFERENTIAL GAME OF PURSUIT WITH INTEGRAL CONSTRAINTS OF A GENERALIZED TYPE

The paper investigates a differential game of simple pursuit, when the controls of two opposing players are subject to integral constraints of a generalized type. The generalization of the proposed restriction lies in the fact that it includes previously known restrictions such as integral, geometric, linear, exponential and their mixtures. In general, it includes 25 types of pursuit problems with such different types of constraints. To solve the pursuit problem under such generalized constraints, we propose a parallel pursuit strategy (Π-strategy for short) and find sufficient conditions for the solvability of this problem. At the end of the article, tables are provided that list each particular type of game, the conditions for its solvability, the resolving function (which determines the corresponding Π-strategy), and the time of capture.

Keywords: differential game, nonlinear integral constraint, pursuer, evader, strategy, pursuit, guaranteed capture time.

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Introduction

A systematic study of conflict-controlled dynamical systems described by differential equations was started in the 50's by the American mathematician R. Isaacs [1], and this theory is referred to by him as "Differential Games". In his monograph [1], a number of applied problems are considered and general ideas are proposed, which are mainly based on game-theoretic and variational methods of solution. Further, on the part of L. D. Berkovitz [2], W. H. Fleming [3], A. Friedman [4] and many other followers, rigorization and development of many of Isaacs' heuristic results were undertaken.

In accordance with the fundamental perspectives in the Theory of Differential Games advanced by L. S. Pontryagin [5], N. N. Krasovskii and A. I. Subbotin [6], a differential game is viewed as a control problem from the standpoint of either a pursuer or an evader. From this viewpoint, the game comes to either a pursuit (convergence) problem or an evasion (escape) problem.

Amid several examples examined in [1], the "Life line" game has a special role as an example of a differential game with phase constraints. For the case when controls of both players are bounded with geometrical constraints, L. A. Petrosyan completely solved this game by means of the "parallel pursuit strategy" (the Π-strategy), which was introduced by him [7]. This strategy later became an efficient method in the solution of other differential games of pursuit (see e. g. B. N. Pshenichnyi and V. V. Ostapenko [8], A. A. Chikrii and A. A. Belousov [9], D. P. Kim [10], N. Yu. Satimov [11], B. B. Rikhsiev [12], A. A. Azamov and B. T. Samatov [13]). The Π-strategy subsequently acted as the starting point for the growth of the pursuit method in differential games with many pursuers (see e. g. B. N. Pshenichnyi [14], B. N. Pshenichnyi, A. A. Chikrii, and J. S. Rappoport [15], A. A. Chikrii [16], L. A. Petrosyan [17, 18], N. N. Petrov [19, 20], A. I. Blagodatskikh and N. N. Petrov [21], N. L. Grigorenko [22], A. A. Azamov [23], B. T. Samatov [24–29]).

In the Theory of Differential Games, control functions are chiefly subjected to geometrical, integral or mixed constraints (A. N. Dar'in and A. B. Kurzhanskii [30], D. V. Kornev and

N. Yu. Lukoyanov [31], G. I. Ibragimov [32, 33]). Problems related to integral constraints are more intriguing and more complex than problems concerned with geometrical constraints. Yet, constraints of both kinds are practically important. Geometrical constraints denote the restricted nature of the dynamical chances of an object (for instance, a restriction on the propulsion). The integral constraints express the limited property of resources (for instance, fuel). In practical applications terms, simple motion differential games where integral and geometrical constraints put on controls at the same time, as well as with diverse constraints, were extensively studied (B. T. Samatov [25], B. T. Samatov and B. I. Juraev [34, 35], A. N. Dar'in and A. B. Kurzhanskii [30], V. V. Ostapenko and I. L. Ryzhkova [36], A. A. Azamov, A. Sh. Kuchkarov and B. T. Samatov [37]). In [38–45], differential games of pursuit with various constraints imposed on control functions of players were considered and sufficient conditions of the pursuit termination were defined. In [24, 28], the notion of linear constraint, which generalizes both geometrical and integral constraints, was introduced and analogues of the Π-strategy were suggested to solve the pursuit problem.

In this paper, we study a differential game of simple pursuit with nonlinear integral constraints on the players' controls. To solve this pursuit problem, a more general Π-strategy is applied and the evader is caught with its help. It should be noted that the constraint on the players' controls proposed in the paper generalizes the previously known constraints, such as integral, geometric, linear, and their combined variants. Accordingly, the strategy and conditions for catching the evader are generalized, which are presented here in the form of tables. The obtained results are based on previously known works as [4, 13, 18, 29–31, 46–49] and extend the studies of R. Isaacs, L. A. Petrosyan, B. N. Pshenichny, A. A. Azamov and others, including the authors.

§ 1. Statement of the problems

Consider a differential game of two players. Let a controlled player P (the pursuer) follow another controlled player E (the evader) in the Euclidean space $\mathbb{R}^n$. Suppose that the positions of the players P and E are described by x and y respectively in $\mathbb{R}^n$, and the players' movements are based on the following differential equations with initial conditions

$$P\colon \dot{x} = u, \quad x(0) = x_0, \tag{1.1}$$

$$E\colon \dot{y} = v, \quad y(0) = y_0 \tag{1.2}$$

correspondingly, where $x, y, u, v \in \mathbb{R}^n$, $n \geq 2$; x_0 and y_0 are the initial states of the objects and it is presumed that $x_0 \neq y_0$; u and v are the velocity controls which function as parameters of equations (1.1) and (1.2). We regard the control parameters u and v as measurable functions $u(\cdot)\colon [0, +\infty) \to \mathbb{R}^n$ and $v(\cdot)\colon [0, +\infty) \to \mathbb{R}^n$, respectively.

The classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ such that satisfy the following conditions

$$\int_0^t |u(s)|^2\, ds \leq \rho_0^2, \quad \rho_0 > 0, \quad t \geq 0, \tag{1.3}$$

$$\int_0^t |v(s)|^2\, ds \leq \sigma_0^2, \quad \sigma_0 > 0, \quad t \geq 0, \tag{1.4}$$

respectively, are denoted by $\mathbb{U}_I$ and $\mathbb{V}_I$, where $\mathbb{U}_I, \mathbb{V}_I \subset L_2[0, +\infty)$. Note that in (1.3) and (1.4) and in further constraints, as the norms of the control vectors u and v in the space $\mathbb{R}^n$, we will consider the usual Euclidean norm, i. e., $|u| = \sqrt{u_1^2 + u_2^2 + \ldots + u_n^2}$, where $u_1, u_2, \ldots, u_n$ are the coordinates of the vector u in the space $\mathbb{R}^n$, and $|v| = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$, where $v_1, v_2, \ldots, v_n$ are the coordinates of the vector v in the same space $\mathbb{R}^n$. Usually, the constraint of type (1.3) (or (1.4), respectively) is called *the integral constraint* (in brief, *the I-constraint*), or *the energy constraint* on the controls (see [9, 11, 13, 23, 24, 26–28, 32, 33]).

The classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ such that satisfy the following conditions

$$\int_0^t |u(s)|^2\,ds \le t\rho_1^2, \quad \rho_1 > 0, \quad t \ge 0, \tag{1.5}$$

$$\int_0^t |v(s)|^2\,ds \le t\sigma_1^2, \quad \sigma_1 > 0, \quad t \ge 0, \tag{1.6}$$

respectively, are denoted by $\mathbb{U}_G$ and $\mathbb{V}_G$, where $\mathbb{U}_G, \mathbb{V}_G \subset L_\infty[0, +\infty)$, in the works [13, 23, 24, 28], and the constraint of type (1.5) (or (1.6), respectively) is called *the geometric constraint* (in brief, *the G-constraint*) on the controls. Here the class $\mathbb{U}_G$ (or $\mathbb{V}_G$, respectively) is wider than the class of measurable functions $u(\cdot)$ (or $v(\cdot)$, respectively) fulfilling the geometric constraint $|u(t)| \le \rho_1$ (or $|v(t)| \le \sigma_1$, respectively) for almost every $t \ge 0$, i. e., in this sense it is less hard (see [1, 5, 7, 13, 17, 18, 22–24, 28]). However, we need to note that both of these classes give almost the same results when solving game problems.

In [13, 24, 28], the classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ satisfying the following conditions

$$\int_0^t |u(s)|^2\,ds \le \rho_1^2 t + \rho_0^2, \quad \rho_0 \ge 0, \quad \rho_1 \ge 0, \quad t \ge 0, \tag{1.7}$$

$$\int_0^t |v(s)|^2\,ds \le \sigma_1^2 t + \sigma_0^2, \quad \sigma_0 \ge 0, \quad \sigma_1 \ge 0, \quad t \ge 0, \tag{1.8}$$

respectively, are denoted by $\mathbb{U}_L$ and $\mathbb{V}_L$. The restriction of the form (1.7) or (1.8) is called *the linear constraint* (briefly, *the L-constraint*).

Furthermore, we denote by $\mathbb{U}^1_{I_{\exp}}$ and $\mathbb{V}^1_{I_{\exp}}$ the classes of all measurable functions $u(\cdot)$ and $v(\cdot)$ such that the following conditions, which were first used and investigated in [29],

$$\int_0^t |u(s)|^2\,ds \le \frac{\rho_1^2}{2k}\left(1 - e^{-2kt}\right), \quad \rho_1 > 0, \quad k > 0, \quad t \ge 0, \tag{1.9}$$

$$\int_0^t |v(s)|^2\,ds \le \frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right), \quad \sigma_1 > 0, \quad k > 0, \quad t \ge 0, \tag{1.10}$$

are satisfied, respectively. We call the restriction of the form (1.9) (or (1.10), respectively) *the exponential constraint from scratch*, or briefly, *the $I^1_{\exp}$-constraint*.

In this work, as a generalization of the constraints (1.3)–(1.10) we will introduce a new type of the integral constraint on the controls of the objects P and E, viz., *an exponential constraint with initial resource* (in brief, *$I^2_{\exp}$-constraint*). For the control functions $u(\cdot)$ and $v(\cdot)$, these constraints have the forms

$$\int_0^t |u(s)|^2\,ds \le \frac{\rho_1^2}{2k}\left(1 - e^{-2kt}\right) + \rho_0^2, \quad \rho_0 > 0, \quad \rho_1 > 0, \quad k > 0, \quad t \ge 0, \tag{1.11}$$

$$\int_0^t |v(s)|^2\,ds \le \frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right) + \sigma_0^2, \quad \sigma_0 > 0, \quad \sigma_1 > 0, \quad k > 0, \quad t \ge 0. \tag{1.12}$$

Let the classes of all control functions $u(\cdot)$ and $v(\cdot)$ fulfilling $I^2_{\exp}$-constraints (1.11) and (1.12), respectively be denoted by $\mathbb{U}^2_{I_{\exp}}$ and $\mathbb{V}^2_{I_{\exp}}$. In the constraints (1.3), (1.4), (1.7), (1.8), (1.11), (1.12), the parameters ρ_0^2 and σ_0^2 represent the initial resources of the objects P and E correspondingly.

We need to remark that $I^2_{\exp}$-constraints (1.11) and (1.12) can be looked on as the general form of integral, linear and mixed constraints (see [13, 23–28, 31, 34–36]). Problems with linear and mixed constraints are more complicated than problems with integral and geometric constraints.

Depending on the above-mentioned constraints on the controls of the Pursuer and the Evader, we have twenty five variants of the pursuit–evasion differential games with simple motions (1.1) and (1.2). For definiteness and simplicity, let us call them as follows:

(1) *The* $I^1_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}^1_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}^1_{I_{\exp}}$;

(2) *The* I*-Game* if $u(\cdot) \in \mathbb{U}_I$ and $v(\cdot) \in \mathbb{V}_I$;

(3) *The* $I^1_{\exp}I$*-Game* if $u(\cdot) \in \mathbb{U}^1_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}_I$;

(4) *The* $I^1_{\exp}I^2_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}^1_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$;

(5) *The* $I^2_{\exp}I$*-Game* if $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}_I$;

(6) *The* $I^2_{\exp}I^1_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}^1_{I_{\exp}}$;

(7) *The* $II^2_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}_I$ and $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$;

(8) *The* $II^1_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}_I$ and $v(\cdot) \in \mathbb{V}^1_{I_{\exp}}$;

(9) *The* L*-Game* if $u(\cdot) \in \mathbb{U}_L$ and $v(\cdot) \in \mathbb{V}_L$;

(10) *The* GL*-Game* if $u(\cdot) \in \mathbb{U}_G$ and $v(\cdot) \in \mathbb{V}_L$;

(11) *The* LI*-Game* if $u(\cdot) \in \mathbb{U}_L$ and $v(\cdot) \in \mathbb{V}_I$;

(12) *The* LG*-Game* if $u(\cdot) \in \mathbb{U}_L$ and $v(\cdot) \in \mathbb{V}_G$;

(13) *The* G*-Game* if $u(\cdot) \in \mathbb{U}_G$ and $v(\cdot) \in \mathbb{V}_G$;

(14) *The* GI*-Game* if $u(\cdot) \in \mathbb{U}_G$ and $v(\cdot) \in \mathbb{V}_I$;

(15) *The* IL*-Game* if $u(\cdot) \in \mathbb{U}_I$ and $v(\cdot) \in \mathbb{V}_L$;

(16) *The* IG*-Game* if $u(\cdot) \in \mathbb{U}_I$ and $v(\cdot) \in \mathbb{V}_G$;

(17) *The* $I^2_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$;

(18) *The* $I^2_{\exp}G$*-Game* if $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}_G$;

(19) *The* $I^2_{\exp}L$*-Game* if $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}_L$;

(20) *The* $GI^2_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}_G$ and $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$;

(21) *The* $LI^2_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}_L$ and $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$;

(22) *The* $I^1_{\exp}G$*-Game* if $u(\cdot) \in \mathbb{U}^1_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}_G$;

(23) *The* $I^1_{\exp}L$*-Game* if $u(\cdot) \in \mathbb{U}^1_{I_{\exp}}$ and $v(\cdot) \in \mathbb{V}_L$;

(24) *The* $GI^1_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}_G$ and $v(\cdot) \in \mathbb{V}^1_{I_{\exp}}$;

(25) *The* $LI^1_{\exp}$*-Game* if $u(\cdot) \in \mathbb{U}_L$ and $v(\cdot) \in \mathbb{V}^1_{I_{\exp}}$.

In the present, we are mainly going to investigate the $I^2_{\exp}$-Game of Pursuit and show our main results. The rest twenty four cases will be reviewed in the form of a table in the third section of this work.

Definition 1. The functions $u(\cdot) = (u_1(\cdot), \dots, u_n(\cdot))$ and $v(\cdot) = (v_1(\cdot), \dots, v_n(\cdot))$ are said to be *admissible control functions* of the Pursuer and the Evader, respectively, if (1.11) and (1.12) are satisfied for $u(\cdot)$ and $v(\cdot)$ correspondingly.

Definition 2. For the pair $(x_0, u(\cdot))$, $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$, the solution of equation (1.1)

$$x(t) = x_0 + \int_0^t u(s)\, ds$$

is called *a trajectory of the Pursuer* in the time interval $[0, +\infty)$.

Definition 3. For the pair $(y_0, v(\cdot))$, $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$ the solution of equation (1.2)

$$y(t) = y_0 + \int_0^t v(s)\, ds$$

is called *a trajectory of the Evader* in the time interval $[0, +\infty)$.

The principal aim of the Pursuer is to capture the Evader, i. e., to reach the equality $x(t) = y(t)$ (the Pursuit game), and meanwhile the Evader struggles not to encounter the Pursuer (the Evasion game), viz., to keep on the relation $x(t) \neq y(t)$ for all $t \geq 0$, and in the opposite case, to delay the time of encounter as long as possible. This is the preliminary statement of the pursuit–evasion problems for the considered game.

Lemma 1. *If* $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$, *then* $x(t) \in S_{r(t)}(x_0)$ *for all* $t \geq 0$, *where* $r(t) = \sqrt{\left(\frac{\rho_1^2}{2k} + \rho_0^2\right) t}$ *and* $S_{r(t)}(x_0)$ *is the closed ball of the space* $\mathbb{R}^n$ *with radius* $r(t)$ *and centered at the point* x_0.

P r o o f. Let $u(\cdot) \in \mathbb{U}^2_{I_{\exp}}$. Then from (1.1), (1.11) and the Cauchy–Bunyakovskii inequality, we can write the following estimations down:

$$|x(t) - x_0| \leq \int_0^t |u(s)| ds \leq \sqrt{t} \sqrt{\int_0^t |u(s)|^2\, ds} \leq \sqrt{t} \sqrt{\frac{\rho_1^2}{2k}(1 - e^{-2kt}) + \rho_0^2} < \sqrt{\left(\frac{\rho_1^2}{2k} + \rho_0^2\right) t},$$

where $t \geq 0$. This finishes the proof. □

Definition 4. We call a function $\boldsymbol{u}_{I^2_{\exp}} : \mathbb{R}_+ \times \mathbb{V}^2_{I_{\exp}} \to \mathbb{U}^2_{I_{\exp}}$ *a strategy of the Pursuer* if:

(a) $\boldsymbol{u}_{I^2_{\exp}}(t, v)$ is Borel measurable with respect to v for every fixed t and besides, it is Lebesgue measurable with respect to t for each fixed v;

(b) the inclusion $\boldsymbol{u}_{I^2_{\exp}}(t, v(\cdot)) \in \mathbb{U}^2_{I_{\exp}}$ holds for each $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$ on a finite time interval $[0, t]$; in this case, the function $\boldsymbol{u}_{I^2_{\exp}}(t, v(\cdot))$ is called *a realization of the strategy* $\boldsymbol{u}_{I^2_{\exp}}(t, v)$;

(c) for every $v_1(\cdot)$, $v_2(\cdot) \in \mathbb{V}^2_{I_{\exp}}$ and for each $t \in [0, +\infty)$, the equality $v_1(s) = v_2(s)$ holds for almost everywhere on $[0, t]$, then $u_1(s) = u_2(s)$ is valid for almost everywhere on $[0, t]$, where $u_i(\cdot) = \boldsymbol{u}_{I^2_{\exp}}(t, v_i(\cdot))$, $i = 1, 2$.

Definition 5. It is said that a strategy $\boldsymbol{u}_{I^2_{\exp}} = \boldsymbol{u}_{I^2_{\exp}}(t, v)$ *guarantees realization of capture* at a finite time $T\left(\boldsymbol{u}_{I^2_{\exp}}\right)$ if the equality $x(\theta) = y(\theta)$ is valid for any control $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$ of the Evader at some $\theta \in \left[0, T\left(\boldsymbol{u}_{I^2_{\exp}}\right)\right]$, where $x(t)$ and $y(t)$ are solutions of the initial value problems

$$\dot{x} = \boldsymbol{u}_{I^2_{\exp}}(t, v(t)), \qquad x(0) = x_0, \qquad t \geq 0,$$
$$\dot{y} = v(t), \qquad y(0) = y_0, \qquad t \geq 0,$$

respectively, and the time $T\left(\boldsymbol{u}_{I^2_{\exp}}\right)$ is generally called *a guaranteed capture time*.

Put $z(t) = x(t) - y(t)$, $z_0 = x_0 - y_0$.

Definition 6. A strategy $\boldsymbol{u}_{I^2_{\exp}} = \boldsymbol{u}_{I^2_{\exp}}(t, v)$ is called *a parallel pursuit strategy*, or simply, $\Pi_{I^2_{\exp}}$*-strategy* if, for each control function $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$, a solution $z(t)$ of the initial value problem

$$\dot{z} = \boldsymbol{u}_{I^2_{\exp}}(t, v(t)) - v(t), \quad z(0) = z_0 \tag{1.13}$$

can be described as

$$z(t) = \Lambda_{I^2_{\exp}}(t, v(\cdot))z_0, \quad \Lambda_{I^2_{\exp}}(0, v(\cdot)) = 1,$$

where $\Lambda_{I^2_{\exp}}(t, v(\cdot))$ is the scalar, continuous, and monotonically decreasing function of t, $t \geq 0$, which is said to be *a convergence function of the objects P and E* in the $I^2_{\exp}$-Game of Pursuit.

§ 2. Solution of the $I^2_{\exp}$-Game of Pursuit

In the $I^2_{\exp}$-Game of Pursuit, we assume that the Pursuer is allowed to know the initial states x_0, y_0, the constants $\rho_0, \sigma_0, \rho_1, \sigma_1, k$ and the value of $v(t)$ at each current time t. Below we will give a definition of the parallel pursuit strategy for the Pursuer.

Definition 7. We call the function

$$\boldsymbol{u}_{I^2_{\exp}}(t, v) = v - \lambda_{I^2_{\exp}}(t, v)\xi_0 \tag{2.1}$$

the $\Pi_{I^2_{\exp}}$*-strategy* in the $I^2_{\exp}$-Game of Pursuit, where

$$\lambda_{I^2_{\exp}}(t, v) = \mu_0 + \langle v, \xi_0\rangle + \sqrt{(\mu_0 + \langle v, \xi_0\rangle)^2 + \delta_1 e^{-2kt}}, \tag{2.2}$$

$\mu_0 = \delta_0/2|z_0|$, $\delta_0 = \rho_0^2 - \sigma_0^2$, $\delta_1 = \rho_1^2 - \sigma_1^2$, $\xi_0 = z_0/|z_0|$, $z_0 = x_0 - y_0$, and $\langle v, \xi_0\rangle$ means the inner product of the vectors v and ξ_0 in $\mathbb{R}^n$.

It is worth noting that the function $\lambda_{I^2_{\exp}}(t, v)$ is usually called *the resolving function*.

Proposition 1. *If $\delta_1 \geq 0$, then for any pair $(t, v)\colon \mathbb{R}_+ \times \mathbb{V}^2_{I_{\exp}}$:*

(a) *the function $\lambda_{I^2_{\exp}}(t, v)$ is defined and non-negative;*

(b) *the equality*

$$\left|\boldsymbol{u}_{I^2_{\exp}}(t, v)\right|^2 = |v|^2 + \frac{\delta_0}{|z_0|}\lambda_{I^2_{\exp}}(t, v) + \delta_1 e^{-2kt}, \quad t \geq 0, \tag{2.3}$$

is satisfied.

Lemma 2. *Let $\delta_1 > 0$ and $\delta_0 \geq 0$ be valid. Then the equation*

$$\sqrt{\Phi(t) + \Psi(t)} - \sqrt{\Phi(t)} = |z_0| \tag{2.4}$$

has at least one positive root with respect to t, $t \geq 0$, where

$$\Phi(t) = t\left(\frac{\sigma_1^2}{2k}\left(1 - e^{-2kt}\right) + \sigma_0^2\right), \quad \Psi(t) = \delta_0 t + \frac{\delta_1}{k^2}\left(1 - e^{-kt}\right)^2.$$

P r o o f. From (2.4) let's write the following function down:

$$\Gamma(t) = \sqrt{\Phi(t) + \Psi(t)} - \sqrt{\Phi(t)} - |z_0|.$$

Consider below some properties of $\Gamma(t)$, i. e.,

(a) $\Gamma(0) = -|z_0|$;

(b) take the limit of $\Gamma(t)$ as $t \to +\infty$

$$\lim_{t\to+\infty} \Gamma(t) = \lim_{t\to+\infty} \left[\sqrt{\Phi(t) + \Psi(t)} - \sqrt{\Phi(t)} - |z_0|\right] =$$
$$= \lim_{t\to+\infty} \left[\frac{\Psi(t)}{\sqrt{\Phi(t) + \Psi(t)} + \sqrt{\Phi(t)}} - |z_0|\right] = +\infty.$$

According to the above properties of $\Gamma(t)$, there can be found some $\tau \in [0, +\infty)$ that generates $\Gamma(\tau) = 0$. This completes the proof. □

Definition 8. In the $I^2_{\exp}$-Game of Pursuit, we call the smallest positive root of (2.4) *the guaranteed capture time* and denote it by $T_{I^2_{\exp}}$.

Now we can move on to formulating our main result for the $I^2_{\exp}$-Game of Pursuit.

Theorem 1. *Let* $\delta_1 > 0$ *and* $\delta_0 \geq 0$. *Then* $\Pi_{I^2_{\exp}}$*-strategy* (2.1) *guarantees realization of capture on the time interval* $\left[0, T_{I^2_{\exp}}\right]$.

P r o o f. Assume that the Evader chooses an optional control $v(\cdot) \in \mathbb{V}^2_{I_{\exp}}$. Then the Pursuer applies the $\Pi_{I^2_{\exp}}$-strategy. From (1.13) and (2.1) it proceeds Cauchy's problem

$$\dot{z} = \boldsymbol{u}_{I^2_{\exp}}(t, v(t)) - v(t) = -\lambda_{I^2_{\exp}}(t, v(t))\xi_0, \quad z(0) = z_0, \tag{2.5}$$

where $z = x - y$. Integrating both sides of (2.5) and taking (2.2) into account we find the solution

$$z(t) = \Lambda_{I^2_{\exp}}(t, v(\cdot))z_0, \tag{2.6}$$

where

$$\Lambda_{I^2_{\exp}}(t, v(\cdot)) = 1 - \frac{1}{|z_0|}\int_0^t \lambda_{I^2_{\exp}}(s, v(s))\, ds.$$

Now, analyze the nature of decay of the convergence function $\Lambda_{I^2_{\exp}}(t, v(\cdot))$ with respect to t. Obviously, the function $\Lambda_{I^2_{\exp}}(t, v(\cdot))$ is continuous and monotonically decreasing with respect to t, $t \geq 0$, and therefore, in view of (2.2), the following estimation holds:

$$\Lambda(t, v(\cdot)) \leq 1 - \frac{1}{|z_0|}\int_0^t \left[\mu_0 - |v(s)| + \sqrt{(\mu_0 - |v(s)|)^2 + \delta_1 e^{-2ks}}\right] ds =$$
$$= 1 - \frac{1}{|z_0|}\int_0^t \left[e^{-ks}\left[e^{ks}(\mu_0 - |v(s)|) + \sqrt{(e^{ks}(\mu_0 - |v(s)|))^2 + \delta_1}\right]\right] ds,$$

or

$$\Lambda_{I^2_{\exp}}(t, v(\cdot)) \leq 1 - \frac{1}{|z_0|}\int_0^t \psi(s)\varphi(w(s))\, ds, \tag{2.7}$$

where $\psi(s) = e^{-ks}$, $w(s) = e^{ks}(\mu_0 - |v(s)|)$ and $\varphi(w) = w + \sqrt{w^2 + \delta_1}$. Since $\varphi(w)$ is a convex function ($\ddot{\varphi}(w) > 0$) with w and ψ: $[0, +\infty) \to (0, 1]$ is integrable, the Jensen inequality

$$\int_0^t \psi(s)\varphi(w(s))\,ds \geq \int_0^t \psi(s)\,ds\;\varphi\left(\frac{\int_0^t \psi(s)w(s)\,ds}{\int_0^t \psi(s)\,ds}\right)$$

can be applied to the integral in (2.7). Then for the right side of this Jensen inequality and from the definitions of the functions $\psi(s), w(s)$ and $\varphi(w)$ we have

$$\int_0^t \psi(s)\,ds\;\varphi\left(\frac{\int_0^t \psi(s)w(s)\,ds}{\int_0^t \psi(s)\,ds}\right) = \int_0^t (\mu_0 - |v(s)|)\,ds + \\ + \sqrt{\left(\int_0^t (\mu_0 - |v(s)|)\,ds\right)^2 + \frac{\delta_1}{k^2}(1 - e^{-kt})^2}.$$

Consequently, from (2.7) we get

$$\Lambda_{I^2_{\exp}}(t, v(\cdot)) \leq 1 - \frac{1}{|z_0|}\left[\mu_0 t - p(t) + \sqrt{(\mu_0 t - p(t))^2 + \delta_1\left(\frac{1 - e^{-kt}}{k}\right)^2}\,\right], \tag{2.8}$$

where $p(t) = \displaystyle\int_0^t |v(s)|\,ds$. The function

$$f(t, p) = \mu_0 t - p + \sqrt{(\mu_0 t - p)^2 + \delta_1\left(\frac{1 - e^{-kt}}{k}\right)^2},$$

which is on the right-hand side of (2.8), is monotonically decreasing with respect to p. Taking into account the Cauchy–Bunyakovskii inequality and constraint (1.12), we have

$$\int_0^t |v(s)|\,ds \leq \sqrt{t}\left(\int_0^t |v(s)|^2 ds\right)^{1/2} \leq \sqrt{\Phi(t)}.$$

Then, from these inequalities and from the property of monotonically decreasing function $f(t, p)$, we obtain an inequality of the form

$$f(t, p(t)) \geq \mu_0 t - \sqrt{\Phi(t)} + \sqrt{\left(\mu_0 t - \sqrt{\Phi(t)}\right)^2 + \delta_1\left(\frac{1 - e^{-kt}}{k}\right)^2}$$

for $t \geq 0$. Thus, by virtue of the last inequality, from (2.7) we obtain

$$\Lambda_{I^2_{\exp}}(t, v(\cdot)) \leq \Lambda_{I^2_{\exp}}(t), \tag{2.9}$$

where

$$\Lambda_{I^2_{\exp}}(t) = 1 - \frac{1}{|z_0|}\left[\mu_0 t - \sqrt{\Phi(t)} + \sqrt{\left(\mu_0 t - \sqrt{\Phi(t)}\right)^2 + \delta_1\left(\frac{1 - e^{-kt}}{k}\right)^2}\,\right].$$

Let us show the equivalence of the equalities $\Lambda_{I^2_{\exp}}(t) = 0$ and (2.4). From the latter and from the form $\Lambda_{I^2_{\exp}}(t)$, it easily follows that

$$\sqrt{\left(\mu_0 t - \sqrt{\Phi(t)}\right)^2 + \delta_1\left(\frac{1 - e^{-kt}}{k}\right)^2} + \mu_0 t - \sqrt{\Phi(t)} = |z_0|.$$

Multiplying this equality on both sides by the conjugate part of the left side we have

$$\delta_1\left(\frac{1-e^{-kt}}{k}\right)^2 = |z_0|\left[\sqrt{\left(\mu_0 t-\sqrt{\Phi(t)}\right)^2+\delta_1\left(\frac{1-e^{-kt}}{k}\right)^2}-\mu_0 t+\sqrt{\Phi(t)}\right] \Rightarrow$$

$$\Rightarrow \frac{\delta_1}{|z_0|}\left(\frac{1-e^{-kt}}{k}\right)^2+\mu_0 t-\sqrt{\Phi(t)} = \sqrt{\left(\mu_0 t-\sqrt{\Phi(t)}\right)^2+\delta_1\left(\frac{1-e^{-kt}}{k}\right)^2}.$$

Squaring the latter on both sides, we get

$$\delta_1\left(\frac{1-e^{-kt}}{k}\right)^2+2|z_0|\left(\mu_0 t-\sqrt{\Phi(t)}\right)=|z_0|^2.$$

Since $\mu_0=\delta_0/2|z_0|$ (see (2.2)), we find

$$|z_0|^2+2|z_0|\sqrt{\Phi(t)}-\Psi(t)=0$$

whence (2.4).

Now, by virtue of Lemma 2, there exists a time $T_{I^2_{\exp}}$ such that $\Lambda_{I^2_{\exp}}\left(T_{I^2_{\exp}}\right)=0$. Based on (2.9) there exists some time $\eta\in\left[0,T_{I^2_{\exp}}\right]$ which gives the result $\Lambda_{I^2_{\exp}}(\eta,v(\cdot))=0$ and thereby, by virtue of (2.6) we have $z(\eta)=0$, or, more precisely, $x(\eta)=y(\eta)$.

And now, we will have to prove the admissibility of the strategy (2.1) for all $t\in[0,\eta]$. To do this, suppose that the Evader picks an arbitrary control $v(\cdot)\in\mathbb{V}^2_{I_{\exp}}$. Then taking the integral of both sides of (2.3) and considering (1.12) we obtain the relations

$$\int_0^\eta\left|\boldsymbol{u}_{I^2_{\exp}}(s,v(s))\right|^2 ds=\int_0^\eta|v(s)|^2\,ds+\frac{\delta_0}{|z_0|}\int_0^\eta\lambda_{I^2_{\exp}}(s,v(s))\,ds+\delta_1\int_0^\eta e^{-2ks}\,ds\le$$
$$\le\frac{\sigma_1^2}{2k}(1-e^{-2k\eta})+\sigma_0^2+\delta_0+\frac{\delta_1}{2k}(1-e^{-2k\eta})\le\frac{\rho_1^2}{2k}(1-e^{-2k\eta})+\rho_0^2.$$

This completes the proof of Theorem 1. □

§ 3. Generality of the $I^2_{\exp}$-Game of Pursuit

To compare the results obtained in the works [13, 24, 25, 28, 29, 34, 35], several cases related to the constraints (1.3)–(1.12) are individually given in the following tables.

Table 1. Types of games and conditions of capture

№	Game	k,ρ_1	k,ρ_0	k,σ_1	k,σ_0	Capture condition	Reference
1	$I^1_{\exp}$	$>0,>0$	$>0,0$	$>0,>0$	$>0,0$	$\rho_1>\sigma_1$	[29]
2	I	$>0,0$	$>0,>0$	$>0,0$	$>0,>0$	$\rho_0>\sigma_0$	[13]
3	$I^1_{\exp}I$	$>0,>0$	$>0,0$	$>0,0$	$>0,>0$	$\rho_1>0$	
4	$I^1_{\exp}I^2_{\exp}$	$>0,>0$	$>0,0$	$>0,>0$	$>0,>0$	$\rho_1>\sigma_1$	
5	$I^2_{\exp}I$	$>0,>0$	$>0,>0$	$>0,0$	$>0,>0$	$\rho_1>0$	
6	$I^2_{\exp}I^1_{\exp}$	$>0,>0$	$>0,>0$	$>0,>0$	$>0,0$	$\rho_1>\sigma_1$	
7	$II^2_{\exp}$	$>0,0$	$>0,>0$	$>0,>0$	$>0,>0$	$\rho_0^2\ge 2\sigma_0^2+4\|z_0\|\sigma_1$	
8	$II^1_{\exp}$	$>0,0$	$>0,>0$	$>0,>0$	$>0,0$	$\rho_0^2\ge 4\|z_0\|\sigma_1$	

9	L	$+0, >0$	$+0, >0$	$+0, >0$	$+0, >0$	$\rho_1 > \sigma_1$	[24, 28]
10	GL	$+0, >0$	$+0, 0$	$+0, >0$	$+0, >0$	$\rho_1 > \sigma_1$	[24, 28]
11	LI	$+0, >0$	$+0, >0$	$+0, 0$	$+0, >0$	$\rho_1 > 0$	[13, 28]
12	LG	$+0, >0$	$+0, >0$	$+0, >0$	$+0, 0$	$\rho_1 > \sigma_1$	[13, 24, 28]
13	G	$+0, >0$	$+0, 0$	$+0, >0$	$+0, 0$	$\rho_1 > \sigma_1$	[13]
14	GI	$+0, >0$	$+0, 0$	$+0, 0$	$+0, >0$	$\rho_1 > \sigma_1$	[13, 25]
15	IL	$+0, 0$	$+0, >0$	$+0, >0$	$+0, >0$	$\rho_0^2 \geq 2\sigma_0^2 + 4\|z_0\|\sigma_1$	[13, 28]
16	IG	$+0, 0$	$+0, >0$	$+0, >0$	$+0, 0$	$\rho_0^2 \geq 4\|z_0\|\sigma_1^2$	[13, 25]
17	$I^2_{\exp}$	$>0, >0$	$>0, >0$	$>0, >0$	$>0, >0$	$\rho_1 > \sigma_1$	
18	$I^2_{\exp}G$	$>0, >0$	$>0, >0$	$+0, >0$	$+0, 0$	$\rho_1 > \sigma_1,\ d_1 \geq \|z_0\|$	
19	$I^2_{\exp}L$	$>0, >0$	$>0, >0$	$+0, >0$	$+0, >0$	$\rho_0 > \sigma_0,\ \rho_1 > \sigma_1,\ d_2 \geq \|z_0\|$	
20	$GI^2_{\exp}$	$+0, >0$	$+0, 0$	$>0, >0$	$>0, >0$	$\rho_1 > \sigma_1$	
21	$LI^2_{\exp}$	$+0, >0$	$+0, >0$	$>0, >0$	$>0, >0$	$\rho_0 \geq \sigma_0,\ \rho_1 > \sigma_1$	
22	$I^1_{\exp}G$	$>0, >0$	$>0, 0$	$+0, >0$	$+0, 0$	$\rho_1 > \sigma_1,\ \max d_3 \geq \|z_0\|$	[35]
23	$I^1_{\exp}L$	$>0, >0$	$>0, 0$	$+0, >0$	$+0, >0$	$\rho_1 > \sigma_1,\ \max d_4 \geq \|z_0\|$	
24	$GI^1_{\exp}$	$+0, >0$	$+0, 0$	$>0, >0$	$>0, 0$	$\rho_1 > \sigma_1$	
25	$LI^1_{\exp}$	$+0, >0$	$+0, >0$	$>0, >0$	$>0, 0$	$\rho_1 > \sigma_1$	

where

$$d_1 = \sqrt{\frac{\rho_0^2}{k}\ln\frac{\rho_1}{\sigma_1} + \left(\frac{\sigma_1}{k}\ln\frac{\rho_1}{\sigma_1}\right)^2} - \frac{\sigma_1}{k}\ln\frac{\rho_1}{\sigma_1}, \quad d_2 = \sqrt{\frac{\rho_0^2 - \sigma_0^2}{k}\ln\frac{\rho_1}{\sigma_1} + \left(\frac{\sigma_1}{k}\ln\frac{\rho_1}{\sigma_1}\right)^2} - \frac{\sigma_1}{k}\ln\frac{\rho_1}{\sigma_1},$$

$$d_3 = t(\rho_1 e^{-kt} - \sigma_1), \quad d_4 = \rho_1 e^{-kt} t - \sqrt{\sigma_1^2 t^2 + \sigma_0^2 t},$$

the symbol ">0" means that the corresponding parameter is greater than zero, and the symbol "$+0$" means that the corresponding parameter tends to zero from the positive side.

Table 2. Resolving functions and guaranteed capture times

№	Game	Resolving function	Guaranteed capture time
1	$I^1_{\exp}$	$\langle v, \xi_0\rangle + \sqrt{\langle v, \xi_0\rangle^2 + \delta_1 e^{-2kt}}$	$\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt}) + \frac{\delta_1}{k^2}(1-e^{-kt})^2} - \sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})} = \|z_0\|$
2	I	$\max\left\{0, \frac{\delta_0}{\|z_0\|} + 2\langle v, \xi_0\rangle\right\}$	$\left(\dfrac{\|z_0\|}{\rho_0 - \sigma_0}\right)^2$
3	$I^1_{\exp}I$	$\langle v, \xi_0\rangle - \frac{\sigma_0^2}{2\|z_0\|} + \sqrt{\left(\langle v, \xi_0\rangle - \frac{\sigma_0^2}{2\|z_0\|}\right)^2 + \rho_1^2 e^{-2kt}}$	$\frac{\rho_1}{k}(1-e^{-2kt}) - \sigma_0\sqrt{t} = \|z_0\|$
4	$I^1_{\exp}I^2_{\exp}$	$\langle v, \xi_0\rangle - \frac{\sigma_0^2}{2\|z_0\|} + \sqrt{\left(\langle v, \xi_0\rangle - \frac{\sigma_0^2}{2\|z_0\|}\right)^2 + \delta_1 e^{-2kt}}$	$\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt}) + \frac{\delta_1}{k^2}(1-e^{-kt})^2} - \sqrt{t\left(\frac{\sigma_1^2}{2k}(1-e^{-2kt}) + \sigma_0^2\right)} = \|z_0\|$

5	$I_{\exp}^2 I$	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2+\rho_1^2e^{-2kt}}$	$\sqrt{\rho_0^2t+\frac{\rho_1^2}{k^2}(1-e^{-kt})^2}-\sigma_0\sqrt{t}=\|z_0\|$
6	$I_{\exp}^2 I_{\exp}^1$	$\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}\right)^2+\delta_1e^{-2kt}}$	$\sqrt{t\left(\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\rho_0^2\right)+\frac{\delta_1}{k^2}(1-e^{-kt})^2}-$ $-\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})}=\|z_0\|$
7	$II_{\exp}^2$	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2-\sigma_1^2e^{-2kt}}$	$\frac{2\|z_0\|}{\sqrt{\mu_0^2-4\sigma_1^2}}+\mu_0$
8	$II_{\exp}^1$	$\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}\right)^2-\sigma_1^2e^{-2kt}}$	$\frac{4\|z_0\|^2}{\rho_0^2+\sqrt{\rho_0^2-16\sigma_1^2\|z_0\|^2}}$
9	L	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2+\delta_1}$	$\sqrt{\rho_1^2t^2+\rho_0^2t}-\sqrt{\sigma_1^2t^2+\sigma_0^2t}=\|z_0\|$
10	GL	$\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}\right)^2+\delta_1}$	$\rho_1t-\sqrt{\sigma_1^2t^2+\sigma_0^2t}=\|z_0\|$
11	LI	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2+\rho_1^2}$	$\sqrt{\rho_1^2t^2+\rho_0^2t}-\sigma_0\sqrt{t}=\|z_0\|$
12	LG	$\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}\right)^2+\delta_1}$	$\sqrt{\rho_1^2t^2+\rho_0^2t}-\sigma_1t=\|z_0\|$
13	G	$\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\delta_1}$	$\frac{\|z_0\|}{\rho_0-\sigma_0}$
14	GI	$\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}\right)^2+\rho_1^2}$	$\rho_1t-\sigma_0\sqrt{t}=\|z_0\|$
15	IL	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2-\sigma_1^2}$	$\rho_0\sqrt{t}-\sqrt{\sigma_1^2t^2+\sigma_0^2t}=\|z_0\|$
16	IG	$\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}\right)^2-\sigma_1^2}$	$\rho_0\sqrt{t}-\sigma_1t=\|z_0\|$

17	$I^2_{\exp}$	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2+\delta_1e^{-2kt}}$	$\sqrt{t\left(\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\rho_0^2\right)+\frac{\delta_1}{k^2}(1-e^{-kt})^2}-$ $-\sqrt{t\left(\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\sigma_0^2\right)}=\|z_0\|$
18	$I^2_{\exp}G$	$\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}\right)^2+\rho_1^2e^{-2kt}-\sigma_1^2}$	$\sqrt{t\rho_0^2+t^2\rho_1^2e^{-2kt}}-\sigma_1t=\|z_0\|$
19	$I^2_{\exp}L$	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2+\rho_1^2e^{-2kt}-\sigma_1^2}$	$\sqrt{t^2\rho_1^2e^{-2kt}+\delta_0t}-\sigma_1t=\|z_0\|$
20	$GI^2_{\exp}$	$\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}\right)^2+\rho_1^2-\sigma_1^2e^{-2kt}}$	$\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\delta_1t^2}-$ $-\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\sigma_0^2t}=\|z_0\|$
21	$LI^2_{\exp}$	$\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\delta_0}{2\|z_0\|}\right)^2+\rho_1^2-\sigma_1^2e^{-2kt}}$	$\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\rho_0^2t+\delta_1t^2}-$ $-\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\sigma_0^2t}=\|z_0\|$
22	$I^1_{\exp}G$	$\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\rho_1^2e^{-2kt}-\sigma_1^2}$	$t(\rho_1e^{-kt}-\sigma_1)=\|z_0\|$
23	$I^1_{\exp}L$	$\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle-\frac{\sigma_0^2}{2\|z_0\|}\right)^2+\rho_1^2e^{-2kt}-\sigma_1^2}$	$\rho_1e^{-kt}t-\sqrt{\sigma_1^2t^2+\sigma_0^2t}=\|z_0\|$
24	$GI^1_{\exp}$	$\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\rho_1^2-\sigma_1^2e^{-2kt}}$	$\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\delta_1t^2}-$ $-\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})}=\|z_0\|$
25	$LI^1_{\exp}$	$\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}+$ $+\sqrt{\left(\langle v,\xi_0\rangle+\frac{\rho_0^2}{2\|z_0\|}\right)^2+\rho_1^2-\sigma_1^2e^{-2kt}}$	$\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})+\rho_0^2t+\delta_1t^2}-$ $-\sqrt{t\frac{\sigma_1^2}{2k}(1-e^{-2kt})}=\|z_0\|$

where *the guaranteed capture time* in cases 1, 3–6, 9–12, 14–25 is equal to the first positive root of the given equations.

Conclusion

In the paper, we have investigated the differential game of pursuit with simple motions when the controls of the Pursuer and the Evader are subjected to the integral constraints of a nonlinear type. The zone of reachability of each player at the current time has been presented. To solve the pursuit problem, we have proposed the parallel pursuit strategy (the Π-strategy) for the Pursuer, and sufficient conditions for the capture have been determined. At the end of the paper, it has been shown that the proposed constraint is a generalization of the previously known geometric, integral, and linear constraints on the players' controls. Moreover, in accordance with this work, we will meet a broad scope of game problems for further studies. For instance, differential games of many players with nonlinear integral constraints on the controls of players can be considered in the future.

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ВЕСТНИК УДМУРТСКОГО УНИВЕРСИТЕТА. МАТЕМАТИКА. МЕХАНИКА. КОМПЬЮТЕРНЫЕ НАУКИ
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П-стратегия для дифференциальной игры преследования с интегральными ограничениями обобщенного типа

Ключевые слова: дифферециальные игры, нелинейное интегральное ограничение, преследователь, убегающий, стратегия, преследование, гарантированное время захвата.

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В статье исследуется дифференциальная игра простого преследования, когда на управления двух противоборствующих игроков накладываются интегральные ограничения обобщенного типа. Обобщенность предлагаемого ограничения заключается в том, что оно включает в себя ранее известные ограничения, такие как интегральные, геометрические, линейные, экспоненциальные и их смешанности. В общем, оно включает в себя 25 типов задач преследования с такими разнотипными ограничениями. Для решения задачи преследования при таких обобщенных ограничениях предлагается стратегия параллельного преследования (сокращенно П-стратегия) и находятся достаточные условия разрешимости этой задачи. В конце статьи предлагаются таблицы, где приводятся каждый частный тип игры, условия ее разрешимости, разрешающая функция (определяющая соответствующую П-стратегию) и время поимки.

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