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CORRECT STRUCTURES AND SIMILARITY MEASURES OF SOFT SETS ALONG WITH HISTORIC COMMENTS OF PROF. D. A. MOLODTSOV

After the paper of Molodtsov [Molodtsov D. Soft set theory — First results, *Computers and Mathematics with Applications*, 1999, vol. 37, no. 4–5, pp. 19–31.] first appeared, soft set theory grew at a breakneck pace. Several authors have introduced various operations, relations, results, etc. as well as other aspects in soft set theory and hybrid structures incorrectly, despite their widespread use in mathematics and allied areas. In his paper [Molodtsov D. A. Equivalence and correct operations for soft sets, *International Robotics and Automation Journal*, 2018, vol. 4, no. 1, pp. 18–21.], Molodtsov, the father of soft set theory, pointed out several wrong results and notions. Molodtsov [Molodtsov D. A. Structure of soft sets, *Nechetkie Sistemy i Myagkie Vychisleniya*, 2017, vol. 12, no. 1, pp. 5–18.] also stated that the concept of soft set had not been fully understood and used everywhere. As a result, it is important to revisit the quirks of those conceptions and provide a formal account of the notion of soft set equivalency. Molodtsov already explored many correct operations on soft sets. We use some notions and results of Molodtsov [Molodtsov D. A. Structure of soft sets, *Nechetkie Sistemy i Myagkie Vychisleniya*, 2017, vol. 12, no. 1, pp. 5–18.] to create matrix representations as well as related operations of soft sets, and to quantify the similarity between two soft sets.

Keywords: soft set, operations of soft sets, matrix representation, similarity measure.

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Introduction

Molodtsov [1] introduced soft set theory in 1999. Like fuzzy set theory [2], soft set theory is also a topic of mathematics to deal with uncertainties available in nature. In some aspects, it may look like set-valued analysis [3], but there are many differences between set-valued analysis and soft set theory. It is important to note that soft set theory has contributed to create a new area of analysis named soft rational analysis [4]. But, the idea [5] of soft set theory was developed in 1980. Later, various authors, e. g., [6–14], changed several outcomes that contradicted the accuracy of the ideas and operations established in [1, 5, 15, 16] and others. In [15], Molodtsov mentioned the following: *"...many authors have introduced new operations and relations for soft sets and used these structures in various areas of mathematics and in applied science. Unfortunately in some works the introduction of operations and relationships for soft sets were carried out without due regard to the specific of soft set definition"*. In [16], the notion of correctness of soft operations is explicitly stated. Almost all the authors of the previously stated papers have used the incorrect notion of soft subset of a soft set [15]. Molodtsov also stated in [15] that the authors of [7] established definitions of complement, union, and intersection of soft sets incorrectly. Moreover, many researchers have used these incorrect notions in their works. Çağman and Enginoğlu [10] also defined a new notion of soft set and presented the matrix form of it, which takes into account a subset of the collection of parameters. But, the idea of this incorrect definition is due to Maji et al. [7]. This representation is erroneous because if we are given only matrix representation of a soft set, then we cannot determine the soft set along with original set of attributes. Moreover, this notion has difference with the original notion of soft sets defined by Molodtsov [1]. In this regard, we want to attract attention of the community of soft set researchers to a comment on ResearchGate [17]. On 15/11/2018, Molodtsov commented on a paper of Al-Qudah and Hassan [18], which is available in ResearchGate till writing of this paper. He commented as follows [17]: *"Dear Colleagues, I did not write such a definition of a soft set. You have added one extra condition that the set of parameters is a subset of a fixed set."* We provide screenshot (Fig. 1) of this very important comment in the history of soft set theory for our readers. It was taken by the second author of this paper through his ResearchGate account.

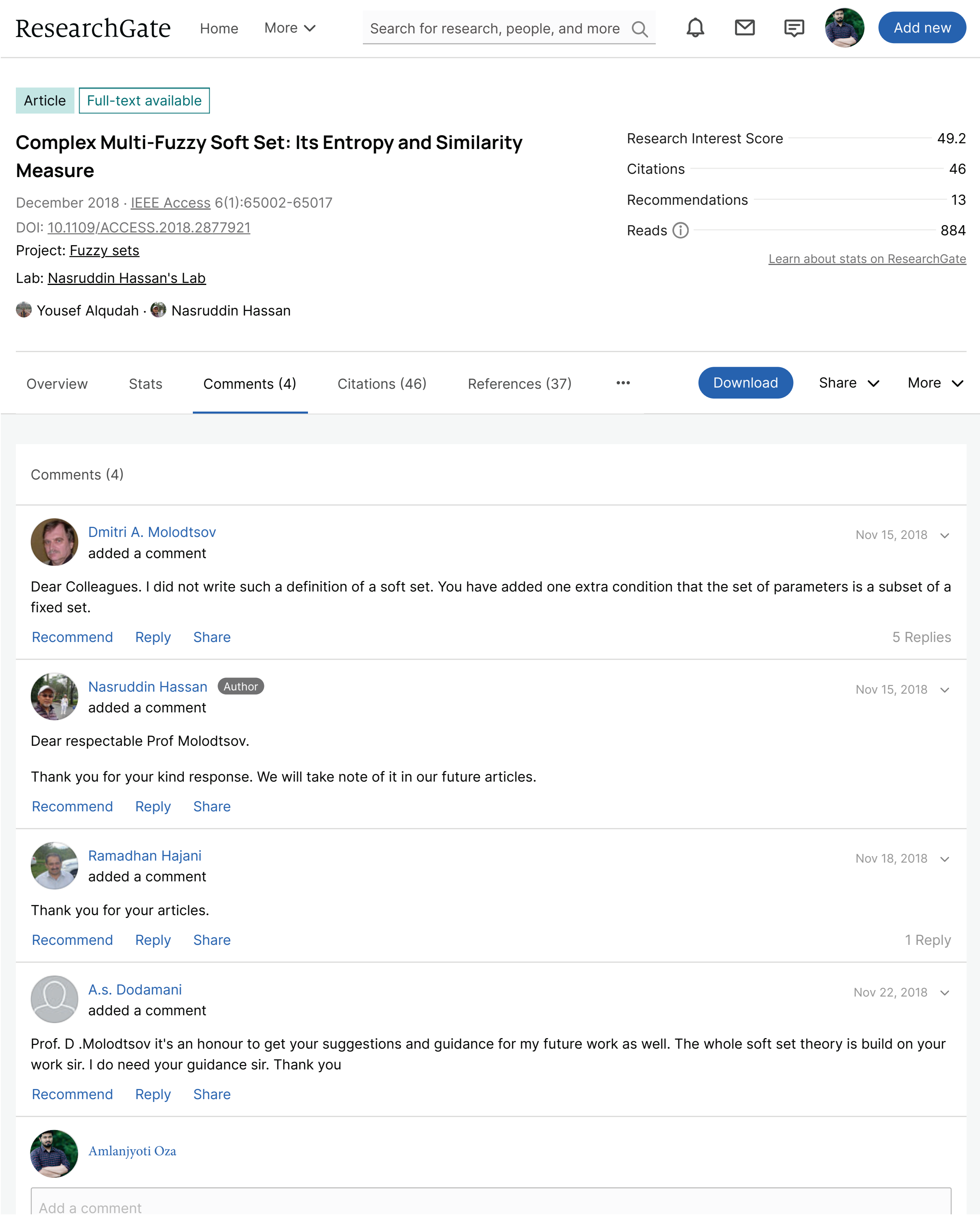


Fig. 1. Comment of Prof. D. Molodtsov on ResearchGate

In [9], the authors considered sets of attributes of the two soft sets for measuring similarity between them. But, it is not necessary that sets of attributes should always contain some common elements. In [13], Kamaci defined a formula for measuring similarity, which results, if there is no common attribute between the two soft sets then the similarity between them is always 0. During early research career in soft set theory and hybrid structures [19,20], the first author of this paper also followed incorrect notions of soft set and associated structures, which were developed by others, not Molodtsov. But, it is not a matter of shame for the first author as he learnt correct concepts of soft set theory and related structures while working with Molodtsov in [4]. Since then, he has been trying to follow correct notions in soft set theory as set by Molodtsov. In [16], Molodtsov mentioned the following: *"The principle of constructing correct operations for soft sets is very simple. Operations should be defined through families of sets* $\tau(S, A)$*"*. Hence, the results of soft sets must be dependent on the sets $\tau(F, A)$ and $\tau(G, B)$ of two soft sets (F, A) and (G, B) respectively, but not on their sets of attributes A and B respectively. In this paper, we try to propose various correct operations of soft sets accurately in matrix forms, as well as we find a similarity measure between two soft sets that is independent of their sets of parameters. We also consider the condition where sets of attributes may have different cardinalities and no shared parameters, which follows philosophy of Molodtsov behind introducing soft set theory [1,5,15,16].

§ 1. Preliminaries

In this section, we discuss some preliminary notions of soft set theory and related results. Let A be a set of parameters and X be a universal set, over which a soft set is defined. The formal definition of a soft set is given bellow.

Definition 1 (see [1]). A pair (S, A) will be called *a soft set over* X, if S is a mapping from the set A to the set of subsets of the set X, i. e., $S\colon A \to 2^X$. In fact, a soft set is a parametrized family of subsets. If the soft set (S, A) is given, then the family $\tau(S, A)$ can be defined as $\tau(S, A) = \{S(a), a \in A\}$ [16].

Definition 2 (see [16]). *Two soft sets* (S, A) and (S', A') given over the universal set X *will be called equal* and we write $(S, A) = (S', A')$ if and only if $S = S'$ and $A = A'$.

Definition 3 (see [16]). *Two soft sets* (S, A) and (S', A') given over the universal set X *will be called equivalent* and written as $(S, A) \cong (S', A')$ if and only if $\tau(S, A) = \tau(S', A')$.

It is easy to note that equivalence of soft set is an equivalence relation.

Definition 4 (see [16]). *A soft set* (S, A) *internally approximates a soft set* (F, D) denoted by $(S, A) \subseteq (F, D)$ if for any $d \in D$ such that $F(d) \neq \phi$, there exists $a \in A$ for which $\phi \neq S(a) \subseteq$ $\subseteq F(d)$.

Definition 5 (see [16]). *A soft set* (S, A) *externally approximates a soft set* (F, D) denoted by $(S, A) \supseteq (F, D)$ if for any $d \in D$ such that $F(d) \neq X$, there exists $a \in A$ for which $X \neq S(a) \supseteq$ $\supseteq F(d)$.

Definition 6 (see [16]). If $(S, A) \subseteq (F, D)$ but the relation $(F, D) \subseteq (S, A)$ has no place, then we say that *the soft set* (S, A) *internally strictly approximate the soft set* (F, D), denoted by $(S, A) \subset (F, D)$.

Definition 7 (see [16]). If $(S, A) \supseteq (F, D)$ but the relation $(F, D) \supseteq (S, A)$ has no place, then we say that *the soft set* (S, A) *externally strictly approximate the soft set* (F, D), denoted by $(S, A) \supset (F, D)$.

Definition 8 (see [16]). *The soft set* (S, A) *is internally equivalent to the soft set* (F, D), denoted by $(S, A) \overset{\subseteq}{\approx} (F, D)$ if $(S, A) \subseteq (F, D)$ and $(F, D) \subseteq (S, A)$.

Definition 9 (see [16]). *The soft set* (S, A) *is externally equivalent to the soft set* (F, D), denoted by $(S, A) \overset{\supseteq}{\approx} (F, D)$ if $(S, A) \supseteq (F, D)$ and $(F, D) \supseteq (S, A)$.

Definition 10 (see [16]). *The soft set* (S, A) *is weakly equivalent to the soft set* (F, D), denoted by $(S, A) \approx (F, D)$ if $(S, A) \overset{\subseteq}{\approx} (F, D)$ and $(S, A) \overset{\supseteq}{\approx} (F, D)$.

Definition 11 (see [16]). The minimal and maximal on inclusion of sets of the family $\tau(S, A)$ is defined as follows:

$$\mathrm{MIN}(\tau(S, A)) = \{B \in \tau(S, A) \mid B \neq \phi,\ \nexists B' \in \tau(S, A) \colon B' \subset B \neq B' \neq \phi\},$$
$$\mathrm{MAX}(\tau(S, A)) = \{B \in \tau(S, A) \mid B \neq X,\ \nexists B' \in \tau(S, A) \colon B' \supset B \neq B' \neq X\}$$

Definition 12 (see [16]). *A relation* Ω *is called correct* if for any quadruple of pairwise equivalent soft sets $(S, A) \cong (S', A')$, $(F, D) \cong (F', D')$, given over the universal set X, the following equality is satisfied:

$$\Omega((S, A), (F, D)) = \Omega((S', A'), (F', D')).$$

Definition 13 (see [16]). *The unary operation complement of* (S, A), $C(S, A) = (W, A)$ is defined as follows: the set of parameters remains the same and the mapping is given by $W(a) = X \setminus S(a)$, for any $a \in A$.

Definition 14 (see [16]). *The binary operation union* $(S, A) \cup (F, D) = (H, A \times D)$ *for a pair of soft sets* (S, A) *and* (F, D) *given over a universal set* X is defined as follows: the parameter set is chosen equal to the direct product of the parameter sets, i. e., equal to $A \times D$, and the corresponding mappings are given by the formula $H(a, d) = S(a) \cup F(d)$, $(a, d) \in A \times D$.

Definition 15 (see [16]). *The binary operation intersection* $(S, A) \cap (F, D) = (W, A \times D)$ *for a pair of soft sets* (S, A) *and* (F, D) *given over a universal set* X is defined as follows: the parameter set is chosen equal to the direct product of the parameter sets, i. e., equal to $A \times D$, and the corresponding mappings are given by the formula $W(a, d) = S(a) \cap F(d)$, $(a, d) \in A \times D$.

Definition 16. *The binary operation product* $(S, A) \times (F, D) = (X, A \times D)$ *for a pair of soft sets* (S, A) *and* (F, D) *given over a universal set* X is defined as follows: the parameter set is chosen equal to the direct product of the parameter sets, i. e., equal to $A \times D$ and the corresponding mappings are given by the formula $X(a, d) = S(a) \times F(d)$, $(a, d) \in A \times D$.

Now, we consider an illustrative example to discuss the above operations.

Example 1. Let us consider two soft sets (F, A) and (G, B) over a universal set X, where A and B be two sets of attributes. We consider $X = \{a, b, c\}$, $A = \{x, y, z\}$, and $B = \{m, n, o\}$. We define the soft set (F, A) as follows:

$$F(x) = \{b, c\}, \quad F(y) = \{c\}, \quad \text{and} \quad F(z) = \{a\}.$$

Hence, $\tau(F, A) = \{\{b, c\}, \{c\}, \{a\}\}$.

Similarly, we define the soft set (G, B) as follows:

$$G(m) = \{a\}, \quad G(n) = \{c\}, \quad \text{and} \quad G(o) = \{c\}.$$

Hence, $\tau(G, B) = \{\{a\}, \{c\}\}$.

Now, we consider the unary operation complement C of the soft set (F,A) which is given by $\mathrm{C}(F,A)=(CF,A)$, where $CF(a)=X\setminus S(a), \forall a\in A$. So, $CF(x)=\{a\}$, $CF(y)=\{a,b\}$, and $CF(z)=\{b,c\}$. Therefore, $\tau(CF,A)=\{\{a\},\{a,b\},\{b,c\}\}$.

Again, we consider the binary operation union $\bigcup$ between the soft sets (F,A) and (G,B) denoted by $(F,A)\bigcup(G,B)=(H,A\times B)$, where $A\times B$ is the Cartesian product of A and B, and the mapping is given by $H(a,b)=F(a)\cup G(b)$, where $(a,b)\in A\times B$. Thus, $A\times B=\{\{x,m\},\{x,n\},\{x,o\},\{y,m\},\{y,n\},\{y,o\},\{z,m\},\{z,n\},\{z,o\}\}$. So, we obtain $H(x,m)=\{a,b,c\}$, $H(x,n)=\{b,c\}$, $H(x,o)=\{b,c\}$, $H(y,m)=\{a,c\}$, $H(y,n)=\{c\}$, $H(y,o)=\{c\}$, $H(z,m)=\{a\}$, $H(z,n)=\{a,c\}$, and $H(z,o)=\{a,c\}$. Hence, $\tau(H,A\times B)= =\{\{a,b,c\},\{b,c\},\{a,c\},\{c\},\{a\}\}$.

Now, we also consider the binary operation intersection $\bigcap$ between (F,A) and (G,B) denoted by $(F,A)\bigcap(G,B)=(W,A\times B)$, and the mapping is given by $W(a,b)=F(a)\cap G(b)$, where $(a,b)\in A\times B$. Thus, we get $W(x,m)=\phi$, $W(x,n)=\{c\}$, $W(x,o)=\{c\}$, $W(y,m)=\phi$, $W(y,n)=\{c\}$, $W(y,o)=\{c\}$, $W(z,m)=\{a\}$, $W(z,n)=\phi$, and $W(z,o)=\phi$. Therefore, $\tau(W,A\times B)=\{\{a\},\{c\},\phi\}$.

Also, we consider the binary operation product $\times$ between (F,A) and (G,B), denoted by $(F,A)\times(G,B)=(K,A\times B)$, and the mapping is given by $K(a,b)=F(a)\times G(b)$, where $(a,b)\in A\times B$. Thus, $K(x,m)=\{b,c\}\times\{a\}=\{(b,a),(c,a)\}$, $K(x,n)=\{b,c\}\times\{c\}= =\{(b,c),(c,c)\}$, $K(x,o)=\{b,c\}\times\{c\}=\{(b,c),(c,c)\}$, $K(y,m)=\{c\}\times\{a\}=\{(c,a)\}$, $K(y,n)=\{c\}\times\{c\}=\{(c,c)\}$, $K(y,o)=\{c\}\times\{c\}=\{(c,c)\}$, $K(z,m)=\{a\}\times\{a\}= =\{(a,a)\}$, $K(z,n)=\{a\}\times\{c\}=\{(a,c)\}$, and $K(z,o)=\{a\}\times\{c\}=\{(a,c)\}$. Hence, $\tau(K,A\times B)=\{\{(b,a),(c,a)\},\{(b,c),(c,c)\},\{(c,a)\},\{(c,c)\},\{(a,a)\},\{(a,c)\}\}$.

§ 2. Matrix representations of soft sets

In this section, we discuss the matrix representation of a soft set. A soft set (F,A) defined over a universal set X can be represented by a matrix M such that the number of rows of M is equal to the cardinality of the universal set X, and the number of columns of M is equal to the cardinality of the set of attribute A.

We consider the cardinality of X and the cardinality of A to be finite for the practical feasibility of computation and other real life purposes. Let us consider $|X|=n$ and $|A|=m$, where $X=\{x_1,x_2,x_3,\ldots,x_n\}$ and $A=\{a_1,a_2,a_3,\ldots,a_m\}$. Then, the binary representation table of (F,A) is given below.

	$F(a_1)$	$F(a_2)$	$F(a_3)$	$\ldots$	$F(a_m)$
x_1	m_{11}	m_{12}	m_{13}	$\ldots$	m_{1m}
x_2	m_{21}	m_{22}	m_{23}	$\ldots$	m_{2m}
x_3	m_{31}	m_{32}	m_{33}	$\ldots$	m_{3m}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_n	m_{n1}	m_{n2}	m_{n3}	$\ldots$	m_{nm}

Table 1. Binary representation table of (F,A)

Here, $m_{ij}=\begin{cases}1, & \text{if } x_i\in F(a_j),\\ 0, & \text{if } x_i\notin F(a_j),\end{cases}$ $i=1,2,3,\ldots,n$, and $j=1,2,3,\ldots,m$. Thus, we get the following matrix M as a matrix representation of (F,A) from Table 1.

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & \dots & m_{1m} \\ m_{21} & m_{22} & m_{23} & \dots & m_{2m} \\ m_{31} & m_{32} & m_{33} & \dots & m_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & m_{n3} & \dots & m_{nm} \end{pmatrix}$$

So, every soft set can be transformed to its matrix form and if the matrix or binary representation table of a soft set is given, then we can easily determine the soft set. Here, we should concern about the order of the elements of X in case of matrix representation, because two different orderings of the elements of the set X may lead to two different representations of the same soft set. Thus, different representations of the same soft set may cause difficulties to define various operations related to the matrix representation. Hence, the order of every x_i must be mentioned in case of matrix representation, where $x_i \in X$. Let us consider the following example to illustrate the matrix representation of soft set (F, A).

Example 2. Let (F, A) be a soft set over a universal set X, where $X = \{a, b, c\}$, $A = \{x, y, z\}$, $F(x) = \{b, c\}$, $F(y) = \{c\}$, and $F(z) = \{a\}$. Then, the binary representation table of (F, A) is given below:

	$F(x)$	$F(y)$	$F(z)$
a	0	0	1
b	1	0	0
c	1	1	0

.

Hence, we obtain the following matrix M for the soft set (F, A):

$$M = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}.$$

§ 3. Operations on soft sets and matrix representations

In this section, we formulate four operations viz. complement, union, intersection and product of soft sets in matrix forms.

3.1 Soft complement

Let $M = (x_{ij})_{m\times n}$ be the matrix representation of the soft set (F, A) defined over a universal set X and order of M is $m \times n$. Then, $M' = (x'_{ij})_{m\times n}$ is the matrix representation of $\mathrm{C}(F, A)$, the complement of the soft set (F, A) and it is defined as below:

$$x'_{ij} = \begin{cases} 1, & \text{if } x_{ij} = 0, \\ 0, & \text{if } x_{ij} = 1. \end{cases}$$

Here, $i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$. It is easy to see that the matrix M' is of the same order as the matrix M.

3.2 Soft union

Let $K = (x_{ij})_{m\times n}$ and $L = (y_{ij})_{m\times p}$ be matrix representations of two soft sets (F, A) and (G, B) respectively, defined over a universal set X. Since both soft sets are defined over X, hence K and L may have orders $m \times n$ and $m \times p$ respectively, where m, n and p are positive integers and $|n - p| \geq 0$. Now, we represent union of (F, A) and (G, B) by matrix representation and we consider M to be the matrix representation of $(F, A) \bigcup (G, B) = (H, A \times B)$. Then, the matrix M is of order $m \times (np)$. Let us define the matrix $M = (m_{ij})_{m \times (np)}$ as follows:

$$m_{ij} = \begin{cases} \max\{x_{i1}, y_{ij}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = 1, 2, \dots, p; \\ \max\{x_{i2}, y_{i(j-p)}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = p+1, p+2, \dots, 2p; \\ \max\{x_{i3}, y_{i(j-2p)}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = 2p+1, 2p+2, \dots, 3p; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ \max\{x_{in}, y_{i(j-(n-1)p)}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = (n-1)p+1, \dots, np. \end{cases}$$

The matrix M follows the definition of union of two soft sets (F, A) and (G, B) given in [16].

3.3 Soft intersection

Let $K = (x_{ij})_{m \times n}$ and $L = (y_{ij})_{m \times p}$ be matrix representations of two soft sets (F, A) and (G, B) respectively, defined over a universal set X. Since both soft sets are defined over X, hence K and L may have orders $m \times n$ and $m \times p$ respectively, where m, n and p are positive integers and $|n - p| \geq 0$. Now, we represent intersection of (F, A) and (G, B) by matrix representation and we consider N to be the matrix of $(F, A) \bigcap (G, B) = (H, A \times B)$. Then, the matrix N is of order $m \times (np)$. Let us define the matrix $N = (n_{ij})_{m \times (np)}$ as follows:

$$n_{ij} = \begin{cases} \min\{x_{i1}, y_{ij}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = 1, 2, \dots, p; \\ \min\{x_{i2}, y_{i(j-p)}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = p+1, p+2, \dots, 2p; \\ \min\{x_{i3}, y_{i(j-2p)}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = 2p+1, 2p+2, \dots, 3p; \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ \min\{x_{in}, y_{i(j-(n-1)p)}\}, & \text{where } i = 1, 2, \dots, m, \text{ and } j = (n-1)p+1, \dots, np. \end{cases}$$

The matrix N also follows the definition of intersection of two soft sets (F, A) and (G, B) given in [16].

3.4 Soft product

Let (F, A) and (G, B) be two soft sets defined over a universal set X, where $X = \{x_1, x_2, \dots, x_m\}$, $A = \{e_1, e_2, \dots, e_n\}$, and $B = \{g_1, g_2, \dots, g_p\}$. Then, we define the mapping $\mathrm{prod}\colon (X \times X) \times (A \times B) \to \{0, 1\}$ such that

$$\mathrm{prod}\big((x_k, x_j), (e_i, g_j)\big) = \begin{cases} 1, & \text{if } x_k \in F(e_i) \text{ and } x_j \in G(g_j), \\ 0, & \text{otherwise.} \end{cases}$$

Here, $k = 1, 2, \dots, m$, $i = 1, 2, \dots, n$, and $j = 1, 2, \dots, p$. Then, the matrix representation of $(F, A) \times (G, B)$ is given by $P = (p_{kj})_{m^2 \times (np)}$ such that,

$$p_{kj} = \begin{cases} 1, & \text{if } \mathrm{prod}\big((x_k, x_j), (e_i, g_j)\big) = 1, \\ 0, & \text{if } \mathrm{prod}\big((x_k, x_j), (e_i, g_j)\big) = 0. \end{cases}$$

Now, we consider some examples to illustrate the matrix representations with the operators complement, union, intersection and product of soft sets.

Example 3. From Example 2, the matrix M' of $\mathrm{C}(F, A)$ can be found as shown below:

$$M' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Example 4. Let us consider union of two soft sets (F, A) and (G, B) defined over a universal set X. Let $X = \{a, b, c\}$, $A = \{x, y, z\}$, and $B = \{m, n, o, p\}$. We define the soft set (F, A) as follows: $F(x) = \{b, c\}$, $F(y) = \{c\}$, and $F(z) = \{a\}$. Similarly, we define the soft set (G, B) as follows: $G(m) = \{a\}$, $G(n) = \{c\}$, $G(o) = \{c\}$, and $G(p) = \{a, c\}$. Then, the set of parameters for $(F, A) \bigcup (G, B)$ is $A \times B = \{\{x, m\}, \{x, n\}, \{x, o\}, \{x, p\}, \{y, m\}, \{y, n\}, \{y, o\}, \{y, p\}, \{z, m\}, \{z, n\}, \{z, o\}, \{z, p\}\}$. Moreover, we have two matrices $M = (x_{ij})_{3\times3}$ and $N = (y_{ij})_{3\times4}$ for (F, A) and (G, B) respectively as shown below:

$$M = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}.$$

Now, the matrix representation of $(F, A) \bigcup (G, B)$ is obtained as a matrix $U = (m_{ij})_{3\times12}$, which is shown below. We can easily calculate m_{ij} for $i = 1, 2, 3$, and $j = 1, 2, 3, 4$. Thus, we obtain the matrix U as shown below:

$$U = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

The matrix U is identical to the matrix obtained by calculating the union of two soft sets (F, A) and (G, B) as defined in Definition 14 and then constructing the matrix from the obtained soft set. Similarly, the intersection between (F, A) and (G, B) can also be calculated using 3.3.

Example 5. Let (F, A) and (G, B) be two soft sets defined over a universal set X, where $X = \{a, b, c\}$, $A = \{m, n\}$, and $B = \{x, y\}$. Let the binary representation tables of (F, A) and (G, B) be M and N respectively:

$$M = \begin{array}{c|cc} & F(m) & F(n) \\ \hline a & 1 & 0 \\ b & 1 & 1 \\ c & 0 & 0 \end{array} \quad \text{and} \quad N = \begin{array}{c|cc} & G(x) & G(y) \\ \hline a & 0 & 0 \\ b & 1 & 0 \\ c & 1 & 1 \end{array}.$$

Then, the binary representation table of $(F, A) \times (G, B) = (H, A \times B)$ is given below:

	$H(m, x)$	$H(m, y)$	$H(n, x)$	$H(n, y)$
(a, a)	0	0	0	0
(a, b)	1	0	0	0
(a, c)	1	1	0	0
(b, a)	0	0	0	0
(b, b)	1	0	1	0
(b, c)	1	1	1	1
(c, a)	0	0	0	0
(c, b)	0	0	0	0
(c, c)	0	0	0	0

Hence, the matrix P of $(F, A) \times (G, B)$ is given by $P = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

Now, we want to mention an important idea related to soft set theory. Molodtsov in [15] wrote: *"Of course, when you specify a soft set, you have some semantic interpretation of this soft set. However, the mathematical formalism of soft sets does not imply any semantic sense on family of subsets or on the parameters. The parameters serve only the purpose to indicate a specific subset . . . To determine topology we have to define only the family of vicinities of a point. No comparison of vicinities and no other properties of these subsets are needed. The situation is quite similar for soft sets, as the soft set is a family of vicinities of a point except that the initial point (as in topology) may not exist. Thus, the role of parameters in definition of soft sets is only auxiliary. Parameters are used only as names of subsets. Therefore, the introduction of the notion of equivalence of soft sets* (S, A) *and* (S', A') *should be based on equality of families of sets* $\tau(S, A)$ *and* $\tau(S', A')$*, but not on equality of point-to-set mappings* S *and* S'*."* Similar assumption can also be found in [5]. Hence, it may be noted that in case of union, intersection and product of soft sets (F, A) and (G, B); the set of parameters $A \times B$ and $B \times A$ are indistinguishable. Hence, we propose the following theorem.

Theorem 1. *Let* (F, A)*,* (G, B) *and* (H, C) *be three soft sets defined over a universal set* X*, then the following properties hold:*

(i) $\mathrm{C}(\mathrm{C}(F, A)) = (F, A)$;

(ii) $(F, A) \bigcup (G, B) = (G, B) \bigcup (F, A)$;

(iii) $\big((F, A) \bigcup (G, B)\big) \bigcup (H, C) = (F, A) \bigcup \big((G, B) \bigcup (H, C)\big)$;

(iv) $(F, A) \bigcap (G, B) = (G, B) \bigcap (F, A)$;

(v) $\big((F, A) \bigcap (G, B)\big) \bigcap (H, C) = (F, A) \bigcap \big((G, B) \bigcap (H, C)\big)$.

P r o o f. We only prove (i) and (ii).

(i) For the complement $\mathrm{C}(F, A) = (W, A)$ of (F, A), the set of parameters remains the same and the mapping is given by $W(a) = X \setminus F(a)$, for $a \in A$. Again, taking complement of the set (W, A), we get $\mathrm{C}(W, A) = (U, A)$ and the mapping is given by $U(a) = X \setminus W(a) = X \setminus \{X \setminus F(a)\} = F(a)$, for $a \in A$. Thus, $\mathrm{C}(\mathrm{C}(F, A)) = (F, A)$.

(ii) Let $(F, A) \cup (G, B) = (H, A \times B)$. Then, the set of parameters for $(F, A) \cup (G, B)$ is $A \times B$, and the corresponding mapping is given by $H(a, b) = F(a) \cup G(b)$, where $(a, b) \in A \times B$. Again, we consider $(G, B) \cup (F, A) = (I, B \times A)$. In this case, the set of parameters is $B \times A$, and the corresponding mapping is given by $I(b, a) = G(b) \cup F(a)$. Since, according to Molodtsov [2], the set of parameters is auxiliary in case of soft set, thus $A \times B$ and $B \times A$ are indistinguishable in the sense that $(a, b) \in A \times B$ and $(b, a) \in B \times A$ are indistinguishable. So, $H(a, b) = I(b, a)$ because $F(a) \bigcap G(b) = G(b) \bigcap F(a)$. Hence, $(F, A) \cup (G, B) = (G, B) \cup (F, A)$. □

§ 4. Similarity between two soft sets

In this section, we define similarity measure between two soft sets (F, A) and (G, B) defined over a universal set X. We consider the concept of matrix representation of a soft set while measuring the similarity between two soft sets. For real world phenomena, we consider both the universal set and the set of parameters of a soft set to be finite.

Definition 17. Let X be a universal set, A and B be two sets of attributes such that $|X| = m$, $|A| = n$ and $|B| = p$. Here, $|K|$ denotes the cardinality of a set K. Without loss of generality, we consider $n \geq p$. Also, we assume $Y = (y_{ij})_{m\times n}$ and $Z = (z_{ij})_{m\times p}$ to be matrix representations of (F, A) and (G, B) respectively. Now, we consider the following two cases.

Case 1: when $n = p$.

If $n = p$, then we construct a new matrix $D = (d_{ij})_{m\times n}$ which depends on matrices Y and Z in the following way:

$$d_{ij} = \begin{cases} 1, & \text{if } y_{ij} = z_{ij}, \\ 0, & \text{if } y_{ij} \neq z_{ij}. \end{cases}$$

Then, *similarity measure between soft sets* (F, A) *and* (G, B) is denoted by $\mathrm{Sim}\{(F, A), (G, B)\}$ and defined as:

$$\mathrm{Sim}\big((F, A), (G, B)\big) = \frac{\sum_{i,j}\{d_{ij} \colon d_{ij} = 1\}}{|X| \times |A|}.$$

Case 2: when $n > p$.

Suppose $n - p = t$, i. e., the matrix $Y = (y_{ij})_{m\times n}$ has exactly t columns more than the matrix $Z = (z_{ij})_{m\times p}$. Now, we first consider the submatrix $W = (w_{ij})_{m\times p}$ of Y with the first p columns and m rows of Y. Now, the matrices Z and W are of the same order. Again, let us define a new matrix $D = (d_{ij})_{m\times p}$ as follows:

$$d_{ij} = \begin{cases} 1, & \text{if } z_{ij} = w_{ij}, \\ 0, & \text{if } z_{ij} \neq w_{ij}. \end{cases}$$

Next, we consider the remaining submatrix of Y with m rows and $(n - t)$ columns beginning from the $(t + 1)$-th column up to the n-th column. We denote it by $M = (m_{ij})_{m\times(n-t)}$, which is of order $m \times (n - t)$. Now, we construct a zero matrix $N = (n_{ij})_{m\times(n-t)}$, in which all the entries are 0 and is of order $m \times (n - t)$. Again, we define a matrix $C = (c_{ij})_{m\times(n-t)}$ as follows:

$$c_{ij} = \begin{cases} 1, & \text{if } m_{ij} = n_{ij} = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then, *similarity measure between the two soft sets* (F, A) *and* (G, B) is defined as below:

$$\mathrm{Sim}\big((F, A), (G, B)\big) = \frac{\sum_{i,j}\{d_{ij} \colon d_{ij} = 1\} + \sum_{i,j}\{c_{ij} \colon c_{ij} = 1\}}{|X| \times \max\{|A|, |B|\}}.$$

These two similarity measures are independent of the set of attributes chosen and they only depend on the families $\tau(F, A)$ and $\tau(G, B)$. Two soft sets are said to be completely similar if their similarity measure is 1 or completely dissimilar if their similarity measure is 0. Thus, we obtain similarity measures using Molodtsov's ideas [15] as stated above. Let us consider an illustrative example.

Example 6. Let M and N be two matrices of two soft sets (F, A) and (G, B) respectively, where

$$M = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

Thus, $d_{11} = 0$, $d_{12} = 0$, $d_{13} = 1$, $d_{21} = 1$, $d_{22} = 1$, $d_{23} = 1$, $d_{31} = 1$, $d_{32} = 0$, and $d_{33} = 1$.

Now, we construct the following matrix $D = (d_{ij})_{3\times 3}$, where $i = 1, 2, 3$, and $j = 1, 2, 3$:

$$D = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

Next, we construct a zero matrix $R = (r_{ij})_{3\times 1}$, and a column matrix $S = (s_{ij})_{3\times 1}$, each of order 3×1, where $i = 1, 2, 3$, and $j = 1$. It is easy to observe that S is the submatrix of N consisting of 4-th column of N:

$$R = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and} \quad S = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

Thus, $c_{11} = 1$, $c_{21} = 0$, and $c_{31} = 1$. Now, we construct the matrix C as follows:

$$C = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

Hence,

$$\operatorname{Sim}\big((F,A),(G,B)\big) = \frac{\sum_{i,j}\{d_{ij}\colon d_{ij}=1\} + \sum_{i,j}\{c_{ij}\colon c_{ij}=1\}}{|X| \times \max\{|A|,|B|\}} = \frac{2}{3}.$$

Theorem 2. *Let (F, A) and (G, B) be two soft sets over a universal set X. Then, the following results hold:*

(i) $0 \le \operatorname{Sim}\big((F,A),(G,B)\big) \le 1$;

(ii) $\operatorname{Sim}\big((F,A),(G,B)\big) = \operatorname{Sim}\big((G,B),(F,A)\big)$;

(iii) $\operatorname{Sim}\big((F,A),(F,A)\big) = 1$.

P r o o f. (i) Let $|X| = m$, $|A| = n$, and $|B| = p$. Without loss of generality, we consider $n \ge p$. Let $Y = (y_{ij})_{m\times n}$ and $Z = (z_{ij})_{m\times p}$ be matrix representations of (F, A) and (G, B) respectively. We consider the following two cases.

Case 1: when $n = p$. $\sum_{i,j}\{d_{ij}\colon d_{ij}=1\} \le |X| \times |A|$. So, we get $\dfrac{\sum_{i,j}\{d_{ij}\colon d_{ij}=1\}}{|X|\times|A|} \le 1$. Hence, we find that $\operatorname{Sim}\big((F,A),(G,B)\big) \le 1$.

Again, $\sum_{i,j}\{d_{ij}\colon d_{ij}=1\} \ge 0$. It implies $\dfrac{\sum_{i,j}\{d_{ij}\colon d_{ij}=1\}}{|X|\times|A|} \ge 0$. Hence, we get $\operatorname{Sim}\big((F,A),(G,B)\big) \ge 0$.

Case 2: when $n > p$. $\sum_{i,j}\{d_{ij}\colon d_{ij}=1\} + \sum_{i,j}\{c_{ij}\colon c_{ij}=1\} \le |X| \times \max\{|A|,|B|\}$. So we have $\dfrac{\sum_{i,j}\{d_{ij}\colon d_{ij}=1\} + \sum_{i,j}\{c_{ij}\colon c_{ij}=1\}}{|X| \times \max\{|A|,|B|\}} \le 1$. Hence we have $\operatorname{Sim}\big((F,A),(G,B)\big) \le 1$.

Also, $\sum_{i,j}\{d_{ij}\colon d_{ij}=1\} + \sum_{i,j}\{c_{ij}\colon c_{ij}=1\} \ge 0$. Thus, we get

$$\frac{\sum_{i,j}\{d_{ij}\colon d_{ij}=1\} + \sum_{i,j}\{c_{ij}\colon c_{ij}=1\}}{|X| \times \max\{|A|,|B|\}} \ge 0.$$

Hence, it implies that $\operatorname{Sim}\big((F,A),(G,B)\big) \ge 0$.

Thus, combining the above results we get $0 \le \operatorname{Sim}\big((F,A),(G,B)\big) \le 1$.

(ii) Let M and N be matrix representations of (F, A) and (G, B) respectively of order $m \times n$ and $m \times p$, respectively.

If $n = p$, then the result is obvious.

If $n > p$ and $n - p = t$, then we construct a zero matrix K of order $m \times (n - t)$ to obtain the matrix C as defined in Definition 17. If $n < p$ and $p - n = t'$, then similarly, we construct the zero matrix K' of order $m \times (p - t')$ to obtain the matrix C. In both cases, we obtain the same matrix C. Hence, it does not affect if we interchange the order of the soft sets for which we have to calculate the similarity measure.

(iii) Directly follows from Definition 17. □

Theorem 3. *If (F, A) is a soft set defined over a universal set X and (CF, A) denotes the complement of the soft set (F, A), then $\mathrm{Sim}\big((F, A), (CF, A)\big) = 0$.*

P r o o f. The proof is obvious. □

Theorem 4. *Let (F, A) and (G, B) be two soft sets defined over a universal set X. If the mappings F and G are one–one and $(F, A) \cong (G, B)$, then $\mathrm{Sim}\big((F, A), (G, B)\big) = 1$, provided the ordering of attributes is independent of choice.*

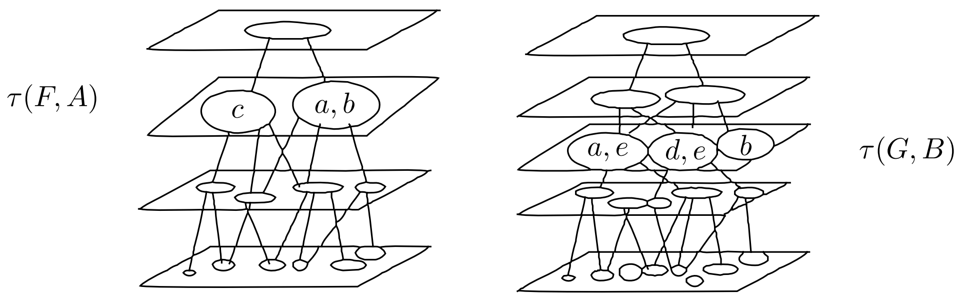
P r o o f. For two soft sets (F, A) and (G, B) defined over a universal set X, $(F, A) \cong (G, B)$ implies $\tau(F, A) = \tau(G, B)$. Since $\tau(F, A) = \tau(G, B)$, then the mappings F and G are one-one implying that $|A| = |B|$. It implies that the matrices of both soft sets will be of the same order. Now, an element of $\tau(F, A)$ represents one column in the respective matrix of (F, A) and similarly we obtain a matrix for (G, B). It is also given that the ordering of attributes is independent of choice. Hence, $\tau(F, A) = \tau(G, B)$ implies matrix representations of soft sets are identical. Thus, $\mathrm{Sim}\big((F, A), (G, B)\big) = 1$. □

Now, we consider a case where $\tau(F, A) \cap \tau(G, B) = \phi$, for two soft sets (F, A) and (G, B). For the practical feasibility, let us discuss a situation by a real life example. Granular computing is an emerging computing paradigm of information processing that concerns the processing of complex information entities called "information granules", which arise in the process of data abstraction and derivation of knowledge from information or data [22]. In this process, there are granules and several granular layers in a granular structure. A granule may be a subset, class, object or cluster of a universe [21]. In granular computing [22, 23], we may construct a granular layer by using a soft set, say (F, A) and granules of the layer may be represented by the elements of $\tau(F, A)$ of the soft set (F, A). Then, we can have similarity measures between the different layers formed by different soft sets. It is often feasible to find that the information in a specific layer of a granular structure does not match to information in an another layer of a different granular structure. In spite of it, we cannot conclude that there is no similarity between the two layers of different granules. For example, we consider two granular layers as shown in Figure 2, represented by two soft sets (F, A) and (G, B) defined over the same universe $X = \{a, b, c\}$. The elements of $\tau(F, A)$ and $\tau(G, B)$ represent granules in two granular layers. Suppose $\tau(F, A) = \{\{a, b\}, \{c\}\}$ and $\tau(G, B) = \{\{a, c\}, \{b\}, \{d, e\}\}$ are representing a set of two granules and a set of three granules in granular structure 1 and granular structure 2 respectively as shown in the Figure 2. Here, $\tau(F, A) \cap \tau(G, B) = \phi$, but we cannot say that there is no similarity between the granular layers because the elements a, b and c are in some granules in both layers. From this discussion, we conclude the following theorem which may be useful for granular computing.

Theorem 5. *If (F, A) and (G, B) are two soft sets defined over a universal set X, then $\tau(F, A) \cap$ $\cap\, \tau(G, B) = \phi$ does not imply that $Sim\big((F, A), (G, B)\big) = 0$ in general.*

Let us consider an example to illustrate the above theorem.

Example 7. Let (F, A) and (G, B) be two soft sets representing two granular layers in two different granular structures defined over a universal set X. Let $X = \{a, b, c, d, e\}$, $A = \{m, n\}$, and $B = \{x, y\}$. The mappings F and G are given by $F(m) = \{a, b\}$, $F(n) = \{e\}$, $G(x) = \{b, c\}$,



Granular strurture 1 Granular strurture 2

Fig. 2. Representations of two granular structures

and $G(y) = \{c, d, e\}$. Hence, $\tau(F, A) = \{\{a, b\}, \{e\}\}$ and $\tau(G, B) = \{\{b, c\}, \{c, d, e\}\}$. In these two families, each element will represent a granule in the granular layer of the respective granular system. We find that $\tau(F, A) \cap \tau(G, B) = \phi$. It is now easy to find that $\mathrm{Sim}\big((F, A), (G, B)\big) = \dfrac{3}{5}$.

Now, let us consider some properties of soft sets with respect to internal and external approximations of soft sets [16].

Definition 18. Let M be a matrix of a soft set (F, A) defined over a universal set X. We consider $A = \{e_1, e_2, \ldots, e_n\}$, i. e., $|A| = n$. Then, *gravity of an attribute* $e_i \in A$ is denoted by $\chi(e_i)$ and it is defined to be the total number of 1's in the ordered i-th column of the matrix M of (F, A), where $i = 1, 2, \ldots, n$.

Theorem 6. *Consider two soft sets (F, A) and (G, B) defined over a universal set X such that $(F, A) \subseteq (G, B)$. Let $|A| = m$ and $|B| = n$. Then, $\chi(e_i) \leq \chi(g_j)$ in some order, where $i = 1, 2, \ldots, m$, and $j = 1, 2, \ldots, n$.*

Corollary 1. *If $(F, A) \subseteq (G, B)$ and $|A| = |B| = n$, then $\chi(e_i) \leq \chi(g_i)$, where $i = 1, 2, \ldots, n$, if the ordering of attributes is independent of choice.*

Let us consider an illustrative example.

Example 8. Let (F, A) and (G, B) be two soft sets defined over a universal set X, where $X = \{a, b, c\}$, $A = \{e_1, e_2, e_3\}$, and $B = \{g_1, g_2, g_3, g_4\}$. If M and N are the matrices of (F, A) and (G, B) respectively, where

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix},$$

then obviously, $(F, A) \subseteq (G, B)$. Hence, $\chi(e_1) = \chi(g_1)$, $\chi(e_2) = \chi(g_4)$, and $\chi(e_3) \leq \chi(g_3)$.

Corollary 2. *Let (F, A) and (G, B) be two soft sets defined over a universal set X and $(F, A) \subseteq$ $\subseteq (G, B)$. Let $A = \{e_1, e_2, \ldots, e_m\}$ and $B = \{g_1, g_2, \ldots, g_n\}$. Then, it is not always true that* $\displaystyle\sum_{i=1}^{m} \chi(e_i) \leq \sum_{j=1}^{n} \chi(g_j)$.

The following example is used to support the preceding corollary.

Example 9. Let (F,A) and (G,B) be two soft sets defined over a universal set X, where $X=\{a,b,c\}$, $A=\{e_1,e_2,e_3\}$, and $B=\{g_1,g_2,g_3,g_4\}$. If M and N are the matrices of (F,A) and (G,B) respectively, where

$$M=\begin{pmatrix}1&1&1\\0&0&0\\1&1&1\end{pmatrix} \quad \text{and} \quad N=\begin{pmatrix}1&1&1&1\\0&0&0&0\\0&1&0&0\end{pmatrix},$$

then obviously, $(F,A)\subseteq(G,B)$. But, $\sum_{i=1}^{3}\chi(e_i)=6\nleq\sum_{j=1}^{5}\chi(g_j)=5$.

Theorem 7. *Let (F,A) and (G,B) be two soft sets defined over a universal set X such that $(F,A)\supseteq(G,B)$, $|A|=m$, and $|B|=n$. Then, $\chi(e_i)\geq\chi(g_j)$ in some orders, where $i=1,2,\ldots,m$, and $j=1,2,\ldots,n$.*

Corollary 3. *If $(F,A)\supseteq(G,B)$ and $|A|=|B|=n$, then $\chi(e_i)\geq\chi(g_i)$, where $i=1,2,\ldots,n$, if the ordering of attributes is independent of choice.*

Corollary 4. *Let (F,A) and (G,B) be two soft sets defined over a universal set X such that $(F,A)\supseteq(G,B)$, $A=\{e_1,e_2,\ldots,e_m\}$, and $B=\{g_1,g_2,\ldots,g_n\}$. Then, it is not always true that $\sum_{i=1}^{m}\chi(e_i)\geq\sum_{j=1}^{n}\chi(g_j)$.*

Theorem 8. *Let (F,A) and (G,A) be two soft sets defined over a universal set X. If $(F,A)\overset{\subseteq}{\approx}(G,A)$, then $\mathrm{Sim}\big((F,A),(G,A)\big)=1$, for some specific orderings of attributes in the matrix representations of two soft sets (F,A) and (G,A).*

P r o o f. Since (F,A) and (G,A) have the same set of attributes, hence their matrix representations are identical. Also, the condition $(F,A)\overset{\subseteq}{\approx}(G,A)$ implies that $(F,A)\subseteq(G,A)$ and $(G,A)\subseteq(F,A)$. Now, due to Theorem 6, we get $\chi(e)_{(F,A)}=\chi(e)_{(G,A)}$, for all $e\in A$. Here, we indicate $\chi(e)_{(F,A)}$ to represent gravity of $e\in A$ in (F,A) and $\chi(e)_{(G,A)}$ to indicate gravity of $e\in A$ in (G,A). Again, since X is ordered while representing the matrices and the ordering of attributes is independent of choice, hence the 1's in each of the matrix will be placed in the same ordered place, which results the same matrices for both soft sets. Thus, we get $\mathrm{Sim}\big((F,A),(G,A)\big)=1$. □

Theorem 9. *Let (F,A) and (G,A) be two soft sets defined over a universal set X. If $(F,A)\overset{\supseteq}{\approx}(G,A)$, then $Sim\big((F,A),(G,A)\big)=1$, for some specific orderings of attributes in the matrix representations of two soft sets (F,A) and (G,A).*

P r o o f. Similar to the above proof. □

Theorem 10. *Let (F,A) and (G,A) be two soft sets defined over a universal set X. We consider M and N to be the matrices of (F,A) and (G,A) respectively, where $|A|=n$. If the columns of M and N are linearly independent and span R^n, then $\mathrm{Sim}\big((F,A),(G,A)\big)=1$, for some specific orderings of attributes in the matrix representations of the soft sets (F,A) and (G,A).*

P r o o f. Since the columns of M and N are linearly independent and span R^n, hence the set of columns of each matrix forms a basis for R^n. Also, since the entries of M and N are 0 and 1 only, hence they will form the identical basis, i. e., the set $\{(1,0,0,\ldots,0),(0,1,0,\ldots,0),\ldots,\ldots,(0,0,0,\ldots,1)\}$. Again, the sets of attributes are the same for (F,A) and (G,A). Hence the matrices will be identical to each other if we arrange the columns in a specific order. Thus, obviously for two identical matrices, we get $\mathrm{Sim}\big((F,A),(G,A)\big)=1$. □

Corollary 5. *For two soft sets (F, A) and (G, A) defined over a universal set X, if F and G are one-one and every element of the families $\tau(F, A)$ and $\tau(G, A)$ belongs to $\mathrm{MIN}(\tau(F, A))$ and $\mathrm{MIN}(\tau(G, A))$ respectively, then $\mathrm{Sim}\big((F, A), (G, A)\big) = 1$, for some specific orderings of attributes in the matrix representations of the soft sets (F, A) and (G, A).*

P r o o f. If every element of the families $\tau(F, A)$ and $\tau(G, A)$ belongs to $\mathrm{MIN}(\tau(F, A))$ and $\mathrm{MIN}(\tau(G, A))$ respectively, then in the matrix representations of (F, A) and (G, B) the set of columns of each matrix is linearly independent, since all the elements of $\tau(F, A)$ and $\tau(G, B)$ are minimal and hence do not contain in any other element of $\tau(F, A)$ and $\tau(G, B)$ respectively. Again, the mappings F and G are one-one. Hence, let $|\tau(F, A)| = |A| = |\tau(G, A)| = n$. Thus, the matrices of (F, A) and (G, B) are of the same order and the linearly independent columns span R^n. Hence, by Theorem 10, $\mathrm{Sim}\big((F, A), (G, A)\big) = 1$. □

Let us consider an example to illustrate the above result.

Example 10. Let (F, A) and (G, A) be two soft sets defined over a universal set X, where $X = \{a, b, c\}$, and $A = \{x, y, z\}$. Now, we define (F, A), where $F(x) = \{b, c\}$, $F(y) = \{c, a\}$, and $F(z) = \{a, b\}$. Hence, $\tau(F, A) = \{\{a, b\}, \{b, c\}, \{c, a\}\}$. Similarly, we define (G, A), where $G(x) = \{a, c\}$, $G(y) = \{a, b\}$, and $G(z) = \{b, c\}$. Hence, $\tau(G, A) = \{\{a, b\}, \{b, c\}, \{c, a\}\}$. Here, the mappings F and G are one-one and all the elements of the families $\tau(F, A)$ and $\tau(G, A)$ belong to $\mathrm{MIN}(\tau(F, A))$ and $\mathrm{MIN}(\tau(G, A))$ respectively. The binary representation tables of (F, A) and (G, A) are given by M' and N' respectively:

$$M' = \begin{array}{c|ccc} & F(x) & F(y) & F(z) \\ \hline a & 0 & 1 & 1 \\ b & 1 & 0 & 1 \\ c & 1 & 1 & 0 \end{array} \quad \text{and} \quad N' = \begin{array}{c|ccc} & G(z) & G(x) & G(y) \\ \hline a & 0 & 1 & 1 \\ b & 1 & 0 & 1 \\ c & 1 & 1 & 0 \end{array}.$$

Thus, we get the following two matrices M and N for (F, A) and (G, A) respectively:

$$M = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Thus, $\mathrm{Sim}\big((F, A), (G, B)\big) = 1$.

Corollary 6. *For two soft sets (F, A) and (G, A) defined over a universal set X, if F and G are one-one and every element of the families $\tau(F, A)$ and $\tau(G, A)$ belongs to $\mathrm{MAX}(\tau(F, A))$ and $\mathrm{MAX}(\tau(G, A))$ respectively, then $\mathrm{Sim}\big((F, A), (G, A)\big) = 1$, for some specific orderings of attributes in the matrix representations of the soft sets (F, A) and (G, A).*

P r o o f. Similar to the above proof. □

§ 5. Decision making using soft sets during smartphone purchase

In this section, we provide a hypothetical decision making scenario of purchasing a smart phone by a customer Mr. Bob. We consider X as the universal set consisting of eight features: *4GB, 6GB, Snapdragon, MediaTek, 64mp, 50mp, 5000mAh, 6000mAh*, i. e.,

$$X = \{\textit{4GB},\ \textit{6GB},\ \textit{Snapdragon},\ \textit{MediaTek},\ \textit{64mp},\ \textit{50mp},\ \textit{5000mAh},\ \textit{6000mAh}\}.$$

Now, Mr. Bob's requirements for a smart phone are based on four attributes: *RAM, Processor, Battery capacity, Camera*. Thus, we consider the set of attributes as A, where

$$A = \{\textit{RAM},\ \textit{Processor},\ \textit{Battery capacity},\ \textit{Camera}\}.$$

His requirements to select a smartphone can be expressed as a soft set (F, A), where $F(\textit{RAM}) = \{\textit{6GB}\}$, $F(\textit{Processor}) = \{\textit{MediaTek}\}$, and $F(\textit{Battery capacity}) = \{\textit{5000mAh}\}$, and

$F(\textit{Camera}) = \{\textit{64mp}\}$. Now, Mr. Bob searches a smartphone named "MOTOROLA G52" in the e-commerce website Flipkart. We denote a soft set (M, A) corresponding to MOTOROLA G52, where $M(\textit{RAM}) = \{\textit{6GB}\}$, $M(\textit{Processor}) = \{\textit{Snapdragon}\}$, $M(\textit{Battery capacity}) = \{\textit{5000mAh}\}$, and $M(\textit{Camera}) = \{\textit{50mp}\}$. Again, he searches another smartphone named "Realme GT Neo 3T" in the official website of Realme. We denote a soft set (R, A) corresponding to Realme GT Neo 3T, where $R(\textit{RAM}) = \{\textit{6GB}\}$, $R(\textit{Processor}) = \{\textit{Snapdragon}\}$, $R(\textit{Battery capacity}) = = \{\textit{5000mAh}\}$, and $R(\textit{Camera}) = \{\textit{64mp}\}$. So, we represent soft sets (F, A), (M, A) and (R, A) in Table 2, Table 3, and Table 4 respectively.

	$F(\textit{RAM})$	$F(\textit{Processor})$	$F(\textit{Battery capacity})$	$F(\textit{Camera})$
4GB	0	0	0	0
6GB	1	0	0	0
Snapdragon	0	0	0	0
MediaTek	0	1	0	0
64mp	0	0	0	1
50mp	0	0	0	0
5000mAh	0	0	1	0
6000mAh	0	0	0	0

Table 2. Binary representation table of (F, A)

	$M(\textit{RAM})$	$M(\textit{Processor})$	$M(\textit{Battery capacity})$	$M(\textit{Camera})$
4GB	0	0	0	0
6GB	1	0	0	0
Snapdragon	0	1	0	0
MediaTek	0	0	0	0
64mp	0	0	0	0
50mp	0	0	0	1
5000mAh	0	0	1	0
6000mAh	0	0	0	0

Table 3. Binary representation table of (M, A)

	$R(\textit{RAM})$	$R(\textit{Processor})$	$R(\textit{Battery capacity})$	$R(\textit{Camera})$
4GB	0	0	0	0
6GB	1	0	0	0
Snapdragon	0	1	0	0
MediaTek	0	0	0	0
64mp	0	0	0	1
50mp	0	0	0	0
5000mAh	0	0	1	0
6000mAh	0	0	0	0

Table 4. Binary representation table of (R, A)

Now, we can find that $\mathrm{Sim}\big((F,A),(M,A)\big) = \dfrac{30}{32}$ and $\mathrm{Sim}\big((F,A),(R,A)\big) = \dfrac{31}{32}$. Hence, Mr. Bob will prefer Realme GT Neo 3T more than MOTOROLA G52.

§ 6. Discussion

The theory of soft set has been applied wrongly to many areas of mathematics and allied areas by almost all the researchers of soft set theory except Molodtsov [15, 16]. As he stated several times [15, 16], correctness of soft set theory is required to maintain the genuine philosophy of soft set theory's importance. Thus, we have tried to establish some results and notions based on the correct structures introduced by only Molodtsov [1, 5, 15, 16]. Similarity measure discussed here may be applied in granular computing as well as in other computing paradigm. Based on our present study, we propose the following conjecture.

Conjecture. *If a relation Ω is correct for any quadruple of pairwise equivalent soft sets $(F,A) \cong$ $\cong (F',A')$ and $(G,B) \cong (G',B')$, defined over a universal set X, then it is not necessary that* $\mathrm{Sim}\big((F,A),(G,B)\big) = \mathrm{Sim}\big((F',A'),(G',B')\big)$.

Not only we have tried to follow correct notions and philosophy of soft set theory, but also several pioneers of fuzzy set theory corrected several misconceptions related to fuzzy set theory. Fuzzy pioneer George J. Klir and his co-authors [24] pointed out several misconceptions available in literature of philosophy of concepts and fuzzy set theory. But in our case, not only a part of soft set theory is wrong but almost all the fundamental operations of soft sets are completely wrong along with the available notions of soft set by Maji et al. [6, 7], Çağman and Enginoğlu [10], and others. For example, we can raise question on definition of soft topology [25–28] because the definition of empty soft set provided by Molodtsov [16] is completely different than that of the notion of empty soft set available in literature of soft set theory. In case of Molodtsov's definition of empty soft set, an empty set of parameters plays a crucial role, but this idea is absent in the definition of empty soft set provided by Maji et al. [7], Çağman and Enginoğlu [10], and others. Since Molodtsov, the father of soft set theory, raised his concern about incorrect notions, operations and related results of soft set theory, hence it is now prime duty of the community of soft set researchers to look back to the beginning of soft set theory and hybrid structures by following the correct path of Molodtsov. Although there are thousands of published papers available on soft set theory and related areas, but it is not our intention to encourage wrong ideas related to soft set theory and hybrid structures. Thus in this paper, we do not focus to apply our results in different areas but our main intention is to develop correct theories of soft sets by following Molodtsov [1, 5, 15, 16].

§ 7. Conclusion

In this paper, we develop some results based on correct notions of soft sets introduced by Molodtsov [1, 5, 15, 16]. Some unary and binary operations on soft sets are defined correctly in matrix forms. Also, similarity measure between two soft sets (F,A) and (G,B) is defined based on the families $\tau(F,A)$ and $\tau(G,B)$, but not on the set of parameters [12, 13]. We hope to continue the process of establishing various ideas related to correct notions of soft set theory by following Molodtsov in our forthcoming papers. We also hope this paper will attract the attention of scientific community related to soft set and hybrid structures, and it will be considered as one of important papers in the history of soft set theory because this paper reports, for the first time, an important comment of Molodtsov on a social platform ResearchGate regarding incorrectness of available definition of soft set along with others.

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С. Ачарджи, А. Оза

Корректные структуры и меры сходства мягких множеств с историческими комментарии профессора Д. А. Молодцова

Ключевые слова: мягкое множество, операции над мягкими множествами, матричное представление, мера сходства.

УДК 510.67, 004.8

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После статьи Молодцова [Molodtsov D. Soft set theory — First results // Computers and Mathematics with Applications. 1999. Vol. 37. No. 4–5. P. 19–31.] теория мягких множеств начала стремительно развиваться. Несколько авторов ввели различные операции, отношения, результаты и т. д., а также другие аспекты в теории мягких множеств и гибридных структур некорректно, несмотря на их широкое применение в математике и смежных областях. В своей работе [Molodtsov D. A. Equivalence and correct operations for soft sets // International Robotics and Automation Journal. 2018. Vol. 4. No. 1. P. 18–21.], Молодцов, отец теории мягких множеств, указал на несколько неверных результатов и понятий. Молодцов [Молодцов Д. А. Структура мягких множеств // Нечеткие системы и мягкие вычисления. 2017. Т. 12. Вып. 1. С. 5–18.] также заявил, что понятие мягкого множества не везде было полностью понято и использовано. В связи с этим важно пересмотреть причуды этих представлений и дать формальное изложение понятия эквивалентности мягкого множества. Молодцов уже исследовал многие корректные операции над мягкими множествами. Мы используем некоторые понятия и результаты Молодцова [Молодцов Д. А. Структура мягких множеств // Нечеткие системы и мягкие вычисления. 2017. Т. 12. Вып. 1. С. 5–18.] для создания матричных представлений, а также связанных с ними операций над мягкими множествами, и для количественной оценки сходства между двумя мягкими множествами.

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